Day 205 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Finding a quadratic from its turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of a quadratic curve.

Find its equation in the form $y = a(x - p)^2 + q$.

The turning point is (1, 2), so $y = a(x - 1)^2 + 2$.

The curve passes through (0, 0):

$0 = a(-1)^2 + 2$, so $-2 = 1\ a$

$a = -2$

Answer: $y = -2(x - 1)^2 + 2$

2 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A systematic sample of 11 patients is taken from a numbered list of 66.

The first one selected is number 1. Find the number of the 8th one selected.

Interval = $66 \div 11 = 6$

Number = $1 + 7 \times 6$

Answer: 43

3 Intercepts of a cubic graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = (x + 5)(x + 2)(x - 4)$. Write down the coordinates of the points where the curve meets the axes.

$y = 0$: $x = -5, -2, 4$

$x = 0$: $y = 5 \times 2 \times (-4) = -40$

Answer: $(-5, 0)$, $(-2, 0)$, $(4, 0)$, $(0, -40)$

4 Pythagorean identity: find cos and tan (All boards)

θ is acute and $\sin\theta = \frac{15}{17}$.

Find the exact values of $\cos\theta$ and $\tan\theta$.

$\sin^2\theta + \cos^2\theta = 1$, so $\cos^2\theta = 1 - \frac{225}{289} = \frac{64}{289}$

θ is acute, so $\cos\theta = \frac{8}{17}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{15}{8}$

Answer: $\cos\theta = \frac{8}{17}$, $\tan\theta = \frac{15}{8}$

5 Smooth inclined plane: acceleration and speed (All boards)

A particle of mass 8 kg slides from rest down a smooth plane inclined at angle α to the horizontal, where $\sin\alpha = \frac{1}{2}$. Take $g = 9.8\ \text{m s}^{-2}$.

Find (a) its acceleration, (b) its speed after 4 s.

(a) Parallel to slope: $mg\sin\alpha = ma$

$a = 9.8 \times \frac{1}{2}$

(b) $v = u + at = 0 + 4.9 \times 4$

Answer: (a) $4.9\ \text{m s}^{-2}$ (b) $19.6\ \text{m s}^{-1}$

6 Exponential graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = 2^x - 8$.

Write down the equation of the asymptote and the coordinates where the curve meets the axes.

As $x \to -\infty$, $2^x \to 0$, so $y \to -8$: asymptote $y = -8$.

$x = 0$: $y = 2^0 - 8 = -7$

$y = 0$: $2^x = 8$, so $2^x = 8$ and $x = 3$

Answer: $y = -8$; $(0, -7)$, $(3, 0)$

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7 Reciprocal trig functions: exact values (All boards)

Write down the exact values of
(a) cot 60° (b) cosec 210°

(a) $\cot 60° = \frac{1}{\tan 60°} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

(b) $\operatorname{cosec} 210° = \frac{1}{\sin 210°} = \frac{1}{-\frac{1}{2}} = -2$

Answer: (a) $\frac{\sqrt{3}}{3}$ (b) -2

8 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is N(μ, 3^2). H_0: $\mu = 35$, H_1: $\mu < 35$.
A sample of 9 has mean $\bar{x} = 32.3$.
Test at the 5% level. Critical value: $z = -1.645$.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (32.3 - 35) \div (3 \div 3)$
$z = -2.70$
-2.70 is inside the critical region, so reject H_0
Answer: $z = -2.70$; reject H_0

9 Particle on a smooth table connected to a hanging particle (All boards)

A 4 kg particle on a smooth horizontal table is joined by a light string over a smooth pulley to a hanging 1 kg particle, as shown. The system is released from rest. Take g = 9.8 m s^{-2}. Find the tension.

Table particle: $T = 4a$
Hanging particle: $1 \times 9.8 - T = 1a$
Adding: $9.8 = 5a$
Answer: 7.84 N

10 Iteration $x_{n+1} = g(x_n)$ (All boards)

$x_{n+1} = \ln(x_n + 3) + 1$, with $x_0 = 2$.
Find x_1, x_2 and x_3 to 3 decimal places.

$x_1 = 2.6094$
$x_2 = 2.7245$
$x_3 = 2.7447$
Answer: $x_1 = 2.609$, $x_2 = 2.724$, $x_3 = 2.745$