Day 204 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:
(a) 4 sin 2x
(b) 4 e^{2x}

(a) dy/dx = 8 cos 2x
(b) dy/dx = 8 e^{2x}
Answer: (a) 8 cos 2x (b) 8 e^{2x}

2 Constant acceleration: time to reach a speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A boat moving in a straight line increases its speed from 8 m s^{-1} to 11 m s^{-1} with constant acceleration 0.5 m s^{-2}. Find the time taken.

v = u + at
11 = 8 + 0.5 t
Answer: 6 s

3 Continuous random variable: probability and mean (CIE)

X has probability density function f(x) = $\frac{1}{9}x^2$ for $0 \le x \le 3$, and 0 otherwise.

Find P(X < 1).

P(X < 1) = $\int$ from 0 to 1 of $\frac{1}{9}x^2$ dx

= $[\frac{1}{27}x^3]$ from 0 to 1

Answer: $\frac{1}{27}$

4 Transforming a point on a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The point P(−4, −6) lies on the curve y = f(x).
Write down the coordinates of the image of P on
(a) y = f(−x) (b) y = f(x + 4)

(a) y = f(−x) changes the sign of x, so P maps to (4, −6).
(b) y = f(x + 4) subtracts 4 from x, so P maps to (−8, −6).
Answer: (a) (4, −6) (b) (−8, −6)

5 Trapezium rule from a function (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 5 strips to estimate

$\int_0^4 2^x\,dx$,

giving your answer to 3 significant figures.

h = $\frac{4-0}{5}$ = 0.8; x = 0, 0.8, 1.6, 2.4, 3.2, 4

y = 1.000, 1.741, 3.031, 5.278, 9.190, 16.00

Area ≈ $\frac{h}{2}$[first + last + 2(sum of the rest)] = $\frac{0.8}{2}$[1.000 + 16.00 + 2 × 19.24]

= 22.2
Answer: 22.2

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6 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 + 4x)^{-1}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots$ with $n = -1$, $y = 4x$

$= 1 + (-1)(4x) + \frac{(-1)((-2))}{2}(4x)^2$

$= 1 - 4x + 16x^2$

Valid when $|4x| < 1$, i.e. $|x| < \frac{1}{4}$

Answer: $1 - 4x + 16x^2$; $|x| < \frac{1}{4}$

7 F = ma: find the force or the mass (All boards)

A resultant force of 12.5 N gives a body an acceleration of 2.5 m s^{-2}. Find the mass of the body.

F = ma

12.5 = m × 2.5

Answer: 5 kg

8 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -4x + k$ is a tangent to the curve

$y = x^2 - 10x + 6$.

Find the value of k.

$x^2 - 10x + 6 = -4x + k$

$x^2 - 6x + (6 - k) = 0$

A tangent touches once: discriminant = 0

$(-6)^2 - 4(6 - k) = 0$

Answer: $k = -3$

9 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a patient takes the drug is 0.5. If so, the probability that the patient recovers is 0.5; otherwise it is 0.4. Given that the patient recovers, find the probability that a patient takes the drug.

P(both) = 0.5 × 0.5 = 0.25; P(not first and second) = 0.5 × 0.4 = 0.2

P(second) = 0.25 + 0.2 = 0.45

P = 0.25 ÷ 0.45

Answer: $\frac{5}{9}$ = 0.556

10 Differentiate tan, sec, cosec, cot (AQA, Edexcel, OCR, MEI, Eduqas)

Differentiate y = cosec 3x with respect to x.

Use the standard result: d/dx(cosec kx) = −k cosec kx cot kx

Answer: dy/dx = −3 cosec 3x cot 3x