Day 203 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Geometric sequences (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In a geometric sequence $u_3 = 45$ and $u_4 = 15$.
Find the first term and the common ratio.

$r = u_4 \div u_3 = 15 \div 45 = \frac{1}{3}$

$a = u_3 \div r^2 = 405$

Answer: a = 405, r = $\frac{1}{3}$

2 Straight line from logarithmic graph (All boards)

Variables x and y satisfy $y = ab^x$. A graph of $\log_{10} y$ against x is a straight line with gradient 0.6 and intercept 0.5.
Find a and b to 3 significant figures.

$\log y = \log a + x \log b$
$\log b = 0.6$ so $b = 10^{0.6} = 3.98$
$\log a = 0.5$ so $a = 10^{0.5} = 3.16$
Answer: a = 3.16, b = 3.98

3 Integrating velocity to find displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line with velocity $v = (6t^2 - 2t + 3)$ m s^{-1} at time t seconds.
Find the distance travelled between t = 2 and t = 4 (the particle does not turn round).

$s = \int v\, dt = 2t^3 - t^2 + 3t$
Distance = $s(4) - s(2)$
$= 124 - 18$
Answer: 106 m

4 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians.
Use the small angle approximations to find an approximation for $\cos\theta + \theta \sin 5\theta$.

$\cos\theta \approx 1 - \frac{\theta^2}{2}$, $\theta \sin 5\theta \approx 5\theta^2$

So $\cos\theta + \theta \sin 5\theta \approx 1 + \frac{9\theta^2}{2}$

Answer: $1 + \frac{9\theta^2}{2}$

5 Free fall: dropped from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A brick is dropped from rest from the top of a cliff of height 78.4 m. Take g = 9.8 m s^{-2} and ignore air resistance. Find (a) the time taken to reach the ground, (b) its speed on hitting the ground.

(a) $s = ut + \frac{1}{2}at^2$: $78.4 = \frac{1}{2} \times 9.8 \times t^2$

$78.4 = 4.9t^2$, so $t^2 = 16$
(b) $v = u + at = 9.8 \times 4$
Answer: (a) 4 s (b) 39.2 m s^{-1}

6 Binomial: at least one success (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Each time a spinner is spun the probability of red is 0.2.
Find the probability of getting red at least once in 5 spins, to 3 s.f.

P(zero) = 0.8^5
P(at least one) = $1 - 0.8^5$
Answer: 0.672

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7 Integration by substitution (All boards)

Use the substitution $u = x^2 + 2$ to find

$\int \frac{3x}{x^2 + 2}\, dx$.

$u = x^2 + 2$, $du = 2x\, dx$, so $x\, dx = \frac{1}{2}\, du$

$\int \frac{3}{2u}\, du = \frac{3}{2} \ln |u|$

Answer: $\frac{3}{2} \ln (x^2 + 2) + c$

8 Real roots and the discriminant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the range of values of k for which $kx^2 + 6x + 8 = 0$ has real roots.

Real roots: $b^2 - 4ac \geq 0$

$36 - 32k \geq 0$, so $k \leq \frac{9}{8}$

Also $k \neq 0$ for the equation to be a quadratic.

Answer: $k \leq \frac{9}{8}$, $k \neq 0$

9 Normal approximation to the binomial (CIE)

X ~ B(40, 0.2). Use a normal approximation to find P(X ≥ 7), to 3 s.f.

np = 8 and npq = 6.4, so use Y ~ N(8, 6.4)
Continuity correction: P(Y > 6.5)
$z = (6.5 - 8) \div \sqrt{6.4}$
Answer: 0.723

10 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Divide $f(x) = 2x^4 + 9x^3 + x^2 - 9x + 12$ by $(x + 4)$.
Find the quotient and the remainder.

Use synthetic division with $a = -4$:
Bottom row: 2, 1, −3, 3, 0
Quotient: $2x^3 + x^2 - 3x + 3$
Remainder: 0
Answer: Quotient $2x^3 + x^2 - 3x + 3$, remainder 0