Day 202 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Arithmetic series: smallest n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic series has first term 8 and common difference 2. Find the smallest n for which the sum of the first n terms exceeds 500.

$S_n = \frac{n}{2}[16 + 2(n - 1)] = \frac{n(2n + 14)}{2}$

$S_{19} = 494$ and $S_{20} = 540$

The sum first exceeds 500 when n = 20.

Answer: n = 20

2 Simplify using identities (All boards)

Simplify $2 - 2\sin^2 x$.

Take out 2: $2(1 - \sin^2 x) = 2\cos^2 x$

Answer: $2\cos^2 x$

3 Position vector under constant velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A boat moves with constant velocity $(-5\mathbf{i} - 12\mathbf{j})$ m s^{-1}.
At time t = 0 its position vector is $(4\mathbf{i} - \mathbf{j})$ m.
(a) Find its position vector when t = 5.
(b) Find the distance it travels in 5 s.

(a) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = (4\mathbf{i} - \mathbf{j}) + 5(-5\mathbf{i} - 12\mathbf{j})$

(b) speed $= \sqrt{5^2 + 12^2} = 13$ m s^{-1}
distance = 13 × 5

Answer: (a) $(-21\mathbf{i} - 61\mathbf{j})$ m (b) 65 m

4 Standard deviation from raw data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Here are 7 values: 16, 20, 4, 8, 16, 4, 14
Find the mean and the standard deviation, to 3 s.f.

$\Sigma x = 82$, $\Sigma x^2 = 1204$, n = 7
Mean = 82 ÷ 7 = 11.71

Variance $= \frac{1204}{7} - (11.71)^2 = 34.78$

Standard deviation $= \sqrt{34.78}$

Answer: mean = 11.7, standard deviation = 5.90

5 Stationary points from a sketch (diagram) (All boards)

The diagram shows the curve $y = x^3 - 3x^2$.
A and B are turning points. Find the coordinates of A and B.

$dy/dx = 3x^2 - 6x$
$dy/dx = 0$ gives x = 0 or x = 2
Substitute: y = 0 at x = 0 and y = −4 at x = 2

Answer: A (0, 0), B (2, −4)

6 Binomial: finding parameters (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(n, p) has mean 12 and variance 3.
Find n and p.

np = 12 and npq = 3
q = 3 ÷ 12 = 0.25, so p = 0.75
n = 12 ÷ 0.75

Answer: n = 16, p = 0.75

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7 Describing transformations of graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of $y = f(x)$ onto the graph of $y = f(x) - 2$.

$y = f(x) - 2$ is obtained from $y = f(x)$ by a:

translation by the vector $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$

Answer: translation by the vector $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$

8 Power of an engine: driving force (CIE)

A car's engine works at a constant rate of 12 kW. Find the driving force when the vehicle is moving at 30 m s^{-1}.

$P = Fv$

$12000 = F \times 30$

Answer: 400 N

9 Integrate using partial fractions (All boards)

$\frac{-x + 2}{(x + 1)(x + 4)} \equiv \frac{A}{x + 1} + \frac{B}{x + 4}$.

Find A and B, and hence find $\int \frac{-x + 2}{(x + 1)(x + 4)}\, dx$.

$-x + 2 \equiv A(x + 4) + B(x + 1)$

$x = -1$ gives $A = 1$; $x = -4$ gives $B = -2$

$\int \left(\frac{1}{x + 1} + \frac{-2}{x + 4}\right) dx = 1 \ln|x + 1| + -2 \ln|x + 4| + c$

Answer: $A = 1$, $B = -2$; $\ln|x + 1| - 2 \ln|x + 4| + c$

10 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = x + k$ is a tangent to the curve

$y = x^2 + 5x + 2$.

Find the value of k.

$x^2 + 5x + 2 = x + k$

$x^2 + 4x + (2 - k) = 0$

A tangent touches once: discriminant = 0

$(4)^2 - 4(2 - k) = 0$

Answer: $k = -2$