Day 20 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 3D vectors: distance, magnitude and midpoint (All boards)

OA = $\begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$ and OB = $\begin{pmatrix} 0 \\ -14 \\ 12 \end{pmatrix}$.

Find AB and |AB|, and a unit vector in the direction of AB.

AB = OB − OA = $\begin{pmatrix} 2 \\ -11 \\ 10 \end{pmatrix}$

$|AB| = \sqrt{2^2 + 11^2 + 10^2} = 15$

Unit vector = $\frac{1}{15}$ AB

Answer: AB = $\begin{pmatrix} 2 \\ -11 \\ 10 \end{pmatrix}$, |AB| = 15, unit vector $\frac{1}{15}\begin{pmatrix} 2 \\ -11 \\ 10 \end{pmatrix}$

2 Finding a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), where f(x) is a cubic. Find f(x) in the form k(x − a)(x − b)(x − c).

The curve crosses the x-axis at x = −3, −1 and 2.
f(x) = k(x + 3)(x + 1)(x − 2)
At (0, −18): k × 3 × 1 × (−2) = −18
k = 3
Answer: f(x) = 3(x + 3)(x + 1)(x − 2)

3 Interpreting the PMCC (AQA, Edexcel, OCR, MEI, Eduqas)

For data set A, r = −0.35. For data set B, r = 0.95.
Which data set has the stronger linear correlation? Give a reason.

Strength depends on the size of r, ignoring the sign.
|−0.35| = 0.35 and |0.95| = 0.95.
Answer: data set B, as |r| is larger

4 Maximum friction on a horizontal surface (All boards)

A box of mass 4 kg rests on rough horizontal ground. The coefficient of friction is 0.5.
Taking g = 9.8 m s^{-2}, find the maximum friction force that can act on the box.

R = mg = 4 × 9.8 = 39.2
F_{max} = μR = 0.5 × 39.2
Answer: 19.6 N

5 Binomial: finding parameters (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(n, p) has mean 12.8 and variance 2.56.
Find n and p.

np = 12.8 and npq = 2.56
q = 2.56 ÷ 12.8 = 0.2, so p = 0.8
n = 12.8 ÷ 0.8
Answer: n = 16, p = 0.8

6 Vector equation of a line (3D) (CIE)

A line has equation r = $\begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}$ + t$\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$.

Show that the point (1, 3, 7) lies on the line.

Equate the first coordinates: 3 + (−1) t = 1, so t = 2
Check the others: −1 + 2 × 2 = 3 and 3 + 2 × 2 = 7
All three agree, so the point is on the line
Answer: Shown (t = 2)

7 Real roots and the discriminant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of k for which
$x^2 + kx + 25 = 0$ has two distinct real roots.

$b^2 - 4ac > 0$
$k^2 - 100 > 0$, so $k^2 > 100$
Answer: $k < -10$ or $k > 10$

8 Integration by substitution (All boards)

Use the substitution u = sin x to find
$\int 2 \cos x \sin^4 x \, dx$.

u = sin x, so du = cos x dx
$\int 2u^4 \, du = \frac{2u^5}{5}$
Replace u by sin x
Answer: $\frac{2\sin^5 x}{5} + c$

9 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $x^2 + 6x$ in the form $(x + p)^2 + q$, and state the coordinates of the minimum point of
$y = x^2 + 6x$.

$x^2 + 6x = (x + 3)^2 - 9$
$x^2 + 6x = (x + 3)^2 - 9$
The minimum is where $(x + 3)^2 = 0$, i.e. $x = -3$
Answer: $(x + 3)^2 - 9$; minimum $(-3, -9)$

10 Constant velocity vector: when does it reach a point? (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves with constant velocity
$(2\mathbf{i} + 5\mathbf{j})$ m s^{-1}. It is at the point with position vector $(\mathbf{i} - 3\mathbf{j})$ m at t = 0. Find the time when its position vector is $(11\mathbf{i} + 22\mathbf{j})$ m.

$r = r_0 + vt$
$(11\mathbf{i} + 22\mathbf{j}) = (\mathbf{i} - 3\mathbf{j}) + t(2\mathbf{i} + 5\mathbf{j})$
Equating i components: $11 = 1 + 2t$
Answer: 5 s