Day 199 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Binomial: P(X = k) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A biased coin lands heads with probability 0.2. It is tossed 14 times. X is the number of heads.
Find P(X = 1), giving your answer to 3 s.f.

$P(X = 1) = {}^{14}C_1 \times 0.2^1 \times 0.8^{13}$

$= 14 \times 0.2^1 \times 0.8^{13}$

Answer: 0.154

2 The remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the remainder when $3x^3 + x^2 + 5x + 3$ is divided by $(x + 4)$.

Remainder = $f(-4)$

$= (-192) + 16 + (-20) + 3$

$= -193$

Answer: -193

3 Velocity at time t with constant acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has initial velocity $(\mathbf{i})$ m s^{-1} and constant acceleration $(-2\mathbf{i} + 4\mathbf{j})$ m s^{-2}.
Find its velocity after 2 s.

$v = u + at$

$v = (\mathbf{i}) + 2(-2\mathbf{i} + 4\mathbf{j})$

$= (\mathbf{i}) + (-4\mathbf{i} + 8\mathbf{j})$

Answer: $(-3\mathbf{i} + 8\mathbf{j})$ m s^{-1}

4 Solving quadratics: surd answers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^2 - 10x - 8 = 0$, giving your answers in the form $p \pm \sqrt{q}$.

Complete the square: $(x - 5)^2 - 25 - 8 = 0$

$(x - 5)^2 = 33$

$x - 5 = \pm\sqrt{33}$

Answer: $x = 5 \pm \sqrt{33}$

5 Collinear points using vectors (All boards)

A(0, 0), B(3, −3) and C(5, −5).
Show that A, B and C are collinear and find AB : BC.

$\text{AB} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$ and $\text{BC} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

$\text{AB} = 3\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\text{BC} = 2\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, so AB and BC are parallel with common point B

Hence A, B, C are collinear and AB : BC = 3 : 2

Answer: collinear; AB : BC = 3 : 2

6 Projectile launched with velocity components (AQA, Edexcel, OCR, MEI)

A stone is projected from level ground with velocity $(14\mathbf{i} + 29.4\mathbf{j})$ m s^{-1} ($\mathbf{i}$ horizontal, $\mathbf{j}$ vertically up). Take g = 9.8 m s^{-2}.
(a) Find the time of flight.
(b) Find the horizontal range.

(a) Vertically: $0 = 29.4\,t - 4.9t^2$

$t = \frac{29.4}{4.9} = 6$

(b) Range = 14×6

Answer: (a) 6 s (b) 84 m

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7 Identity then solve (sin² to cos) (All boards)

Solve $-4\cos^2 x - 6\sin x + 6 = 0$
for $0° \leq x \leq 360°$.

Use $\cos^2 x = 1 - \sin^2 x$: $4\sin^2 x - 6\sin x + 2 = 0$

Factorise and solve: $\sin x = \frac{1}{2}$ or $\sin x = 1$

$x = 30°, 90°, 150°$

Answer: $x = 30°, 90°, 150°$

8 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = 3x + k$ is a tangent to the curve
$y = x^2 - x - 3$.
Find the value of k.

$x^2 - x - 3 = 3x + k$

$x^2 - 4x + (-3 - k) = 0$

A tangent touches once: discriminant = 0

$(-4)^2 - 4(-3 - k) = 0$

Answer: $k = -7$

9 Newton-Raphson: one or two steps (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^2 - 3$.
Use the Newton-Raphson method with $x_0 = 3$ to find x_1,
giving your answer to 3 decimal places.

$f'(x) = 2x$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 3 - \frac{6.000}{6.000} = 2.0000$

Answer: $x_1 = 2.000$

10 Frequency density (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In a histogram, the class 20 to 30 has frequency 20 and is drawn with a bar of height 3 cm.
The class 30 to 35 has frequency 30. Find the height of its bar.

Frequency density = frequency ÷ class width
First bar: 20 ÷ 10 = 2, so 3 cm represents 2
Second bar: 30 ÷ 5 = 6

Height = $3 \times \frac{6}{2}$

Answer: 9 cm