Day 197 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Simplify using identities (All boards)

Simplify $4\sin^2 x + 4\cos^2 x$.

Take out the factor 4: $4(\sin^2 x + \cos^2 x) = 4 \times 1$

Answer: 4

2 Sampling: advantages (AQA, Edexcel, OCR, MEI, Eduqas)

A researcher wants to find out the health of the residents of a village.
Give one advantage and one disadvantage of using quota sampling.

Advantage: no sampling frame is needed and each group is guaranteed to be represented.
Disadvantage: it is not random, so the interviewer may introduce bias.

Answer: Advantage: no sampling frame needed. Disadvantage: not random, interviewer bias

3 Midpoint and length of a line segment (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (4, −3) and B is (4, −9).
Find (a) the midpoint of AB,
(b) the length of AB, as a surd in simplest form.

(a) Midpoint $= \left(\frac{4+4}{2}, \frac{-3+(-9)}{2}\right) = (4, -6)$

(b) $AB^2 = (0)^2 + (-6)^2 = 0 + 36 = 36$

$AB = \sqrt{36} = 6$

Answer: (a) (4, −6) (b) 6

4 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $x^2 + y^2 = 169$.
Find dy/dx in terms of x and y, and hence the gradient at (−5, −12).

Differentiate both sides with respect to x (using dy/dx for y terms)

$\frac{dy}{dx} = -\frac{x}{y}$

At (−5, −12): $\frac{dy}{dx} = -\frac{5}{12}$

Answer: $\frac{dy}{dx} = -\frac{x}{y}$; gradient $-\frac{5}{12}$

5 Integrating acceleration to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its acceleration at time t seconds is $(2t - 3)$ m s^{-2}.
When t = 0 its velocity is 1 m s^{-1}.
Find its velocity when t = 5.

$v = \int a\,dt = t^2 - 3t + c$
$t = 0, v = 1$ gives $c = 1$
When t = 5: $v = 25 + -15 + 1$

Answer: 11 m s^{-1}

6 Integration by substitution (All boards)

Use the substitution $u = \sin x$ to find
$\int 3\cos x \sin^2 x\,dx$.

$u = \sin x$, so $du = \cos x\,dx$

$\int 3u^2\,du = \frac{3u^3}{3}$

Replace u by sin x

Answer: $\frac{3\sin^3 x}{3} + c$

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7 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{1}{x-1} + 2$, $x \neq 1$.

Find the range of f.

$\frac{1}{x-1}$ takes every real value except 0.

Adding 2: f(x) takes every value except 2.

Answer: f(x) takes all real values except 2

8 Work done and kinetic energy (CIE)

A particle of mass 4 kg changes its speed from 5 m s^{-1} to 11 m s^{-1}. Find the increase in its kinetic energy.

$KE = \frac{1}{2}mv^2$

Increase $= \frac{1}{2} \times 4 \times 11^2 - \frac{1}{2} \times 4 \times 5^2$

Answer: 192 J

9 Finding a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), where f(x) is a cubic. Find f(x) in the form $k(x-a)(x-b)(x-c)$.

The curve crosses the x-axis at $x = -4$, 1 and 3.

$f(x) = k(x+4)(x-1)(x-3)$

At (0, 36): $k \times 4 \times (-1) \times (-3) = 36$

$k = 3$

Answer: $f(x) = 3(x+4)(x-1)(x-3)$

10 Mutually exclusive events (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B are mutually exclusive, with P(A) = 0.35 and P(B) = 0.45.

(a) Find $P(A \cup B)$.

(b) Find the probability that neither A nor B happens.

(a) Mutually exclusive, so $P(A \cap B) = 0$

$P(A \cup B) = 0.35 + 0.45 = 0.8$

(b) $1 - 0.8 = 0.2$

Answer: (a) 0.8 (b) 0.2