Day 196 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:

(a) $5\,e^{3x}$

(b) $5 \ln x$

(a) $dy/dx = 15\,e^{3x}$

(b) $dy/dx = \frac{5}{x}$

Answer: (a) $15\,e^{3x}$ (b) $\frac{5}{x}$

2 Differentiating displacement to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A body moves in a straight line. Its displacement from O at time t seconds is s metres, where $s = t^3 - t^2 + 2t + 2$. Find its velocity when t = 2.

$v = \frac{ds}{dt} = 3t^2 - 2t + 2$

When $t = 2$: $v = 3 \times 2^2 + -2 \times 2 + 2$

Answer: 10 m s^{-1}

3 Solve a^x = b using logarithms (All boards)

Solve $5^x = 50$.

Give your answer to 3 significant figures.

Take logs of both sides: $x \log 5 = \log 50$

$x = \frac{\log 50}{\log 5} = 2.43$

Answer: x = 2.43

4 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Use proof by exhaustion to show that $n^3 + 5n$ is divisible by 6 for every integer n with $1 \leq n \leq 5$.

Substitute each value of n:

n = 1: $6 = 6 \times 1$; n = 2: $18 = 6 \times 3$

n = 3: $42 = 6 \times 7$; n = 4: $84 = 6 \times 14$

n = 5: $150 = 6 \times 25$

Every value is a multiple of 6, so the statement is true for $1 \leq n \leq 5$.

Answer: Shown

5 Car towing a caravan (All boards)

A car of mass 1200 kg tows a caravan of mass 400 kg on a level road with acceleration 0.5 m s^{-2}. Resistances: 400 N on the car, 100 N on the caravan. Find the driving force and the tow bar tension.

Whole system: $D - 400 - 100 = 1600 \times 0.5$

Caravan: $T - 100 = 400 \times 0.5$

Answer: D = 1300 N, T = 300 N

6 Finding a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), where f(x) is a cubic. Find f(x) in the form $k(x - a)(x - b)(x - c)$.

The curve crosses the x-axis at $x = -4$, 1 and 3.

$f(x) = k(x + 4)(x - 1)(x - 3)$

At (0, 12): $k \times 4 \times (-1) \times (-3) = 12$

$k = 1$

Answer: f(x) = (x + 4)(x − 1)(x − 3)

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7 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 10^2)$. H_0: $\mu = 115$, H_1: $\mu \neq 115$.

A sample of 16 has mean $\bar{x} = 123.25$.
Test at the 5% level. Critical value: $z = \pm 1.96$.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (123.25 - 115) \div (10 \div 4)$
$z = 3.30$
3.30 is inside the critical region, so reject H_0

Answer: $z = 3.30$; reject H_0

8 Transforming a graph with a labelled turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve $y = f(x)$. The maximum point is shown. Write down the coordinates of the maximum point on the curve with equation
(a) $y = f(x - 3)$ (b) $y = f(x + 3)$

(a) $y = f(x - 3)$: the maximum point moves to $(-1, 2)$.
(b) $y = f(x + 3)$: the maximum point moves to $(-7, 2)$.

Answer: (a) $(-1, 2)$ (b) $(-7, 2)$

9 Geometric distribution (CIE)

A player takes free throws until she scores, with probability 0.5 each time. X is the number of throws.
Find P(X = 2), to 3 s.f.

$P(X = 2) = (1 - p)^1 \times p = (\frac{1}{2})^1 \times \frac{1}{2}$

Answer: 0.250

10 Collinear points using vectors (All boards)

A(−2, 3), B(4, −1) and C(13, k).
Given that A, B and C are collinear, find k.

$AB = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$, $AC = \begin{pmatrix} 15 \\ k - 3 \end{pmatrix}$

$AC = \frac{5}{2}$ AB, so $15 = \frac{5}{2} \times 6$, giving the multiple $\frac{5}{2}$

$k - 3 = \frac{5}{2} \times (-4)$, so $k = -7$

Answer: $k = -7$