Day 193 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Evaluating fractional and negative indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $16^{-\frac{1}{4}}$.

$16^{-\frac{1}{4}} = \frac{1}{16^{\frac{1}{4}}}$

$16^{\frac{1}{4}} = 2$, so $16^{\frac{1}{4}} = 2^1 = 2$

$= \frac{1}{2}$

Answer: $\mathbf{\frac{1}{2}}$

2 Interpreting the PMCC (AQA, Edexcel, OCR, MEI, Eduqas)

The product moment correlation coefficient between hours of sunshine and ice cream sales is $r = -0.85$.
Interpret this value.

r is close to -1, so there is strong negative linear correlation.
As hours of sunshine increases, ice cream sales tends to decrease.
Answer: strong negative linear correlation

3 Position of a point relative to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $(x - 4)^2 + (y + 2)^2 = 49$.
Determine whether the point P$(0, -8)$ is inside, outside or on the circle.

Distance^2 from the centre $(4, -2)$ to P:
$(0 - 4)^2 + (-8 - (-2))^2 = 16 + 36 = 52$
$r^2 = 49$, and $52 > 49$.
Answer: P is outside the circle

4 Exponential growth and decay model (All boards)

A radioactive sample has mass $M = 100\, e^{-0.1t}$ grams after t days.
Find the time for the mass to halve, to 3 significant figures.

$\frac{1}{2} = e^{-0.1t}$

$-0.1t = \ln \frac{1}{2} = -\ln 2$

$t = \frac{\ln 2}{0.1} = 6.93$ days

Answer: t = 6.93 days

5 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Divide $f(x) = 2x^4 - 5x^3 + 7x^2 - 9x - 4$ by $(x - 2)$.
Find the quotient and the remainder.

Use synthetic division with a = 2:
Bottom row: $2, -1, 5, 1, -2$
Quotient: $2x^3 - x^2 + 5x + 1$
Remainder: -2
Answer: Quotient $2x^3 - x^2 + 5x + 1$, remainder -2

6 Binomial: finding parameters (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(20, p)$ and $E(X) = 8$.
Find p and Var(X).

$np = 8$, so $p = 8 \div 20 = 0.4$
$Var(X) = npq = 8 \times 0.6$
Answer: p = 0.4, Var(X) = 4.8

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7 Maximum value from R form (All boards)

f(x) = 9 sin x + 12 cos x + 1.
Write f(x) in the form R sin(x + α) + 1, with α in degrees to 1 d.p., and state the maximum value of f(x).

R = $\sqrt{9^2 + 12^2}$ = 15, tan α = $\frac{12}{9}$ so α = 53.1°

f(x) = 15 sin(x + 53.1°) + 1
sin(x + α) has maximum value 1, so f(max) = 15 + 1
Answer: f(x) = 15 sin(x + 53.1°) + 1; maximum 16

8 Equilibrium of forces given as vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Three forces (2**i** − 4**j**) N, (p**i** + q**j**) N and (−8**i** + 5**j**) N act on a particle which is in equilibrium. **i** and **j** are perpendicular unit vectors. Find the values of p and q.

Sum of forces = 0:
i components: p + (2) + (−8) = 0
j components: q + (−4) + (5) = 0
Answer: p = 6, q = −1

9 Solve basic trig equation (exact, degrees) (All boards)

Solve sin x = $-\frac{1}{2}$ for 0° ≤ x ≤ 360°.

Reference angle: sin 30° = $\frac{1}{2}$

sin is negative in the third and fourth quadrants.
x = 210°, 330°
Answer: x = 210°, 330°

10 Braking: deceleration and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train travelling at 18 m s^{-1} brakes uniformly and comes to rest after travelling 36 m.
(a) Find the deceleration.
(b) Find the time taken to stop.

(a) $v^2 = u^2 + 2as$: $0 = 18^2 + 2 \times a \times 36$
so a = −4.5, a deceleration of 4.5

(b) s = $\frac{(u + v)}{2}$ t: 36 = 9 t

Answer: (a) 4.5 m s^{-2} (b) 4 s