Day 192 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Exponential growth and decay model (All boards)

The number of bacteria is $N = 200\,e^{0.04t}$, where t is the time in hours.
Find (a) N when t = 20, (b) the time when N = 600.
Give answers to 3 s.f.

(a) $N = 200\,e^{0.04 \times 20} = 200\,e^{0.8} = 445$

(b) $e^{0.04t} = 3$, so $0.04t = \ln 3$

$t = \frac{\ln 3}{0.04} = 27.5$ hours

Answer: (a) 445 (b) 27.5 hours

2 Solving quadratics: surd answers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^2 + 2x - 2 = 0$, giving your answers in the form $p \pm \sqrt{q}$.

Complete the square: $(x + 1)^2 - 1 - 2 = 0$

$(x + 1)^2 = 3$

$x + 1 = \pm\sqrt{3}$

Answer: $x = -1 \pm \sqrt{3}$

3 Addition formulae: exact values (All boards)

By writing 15° as 45° − 30°, find the exact value of cos 15°.

$\cos(45° - 30°) = \cos 45° \cos 30° + \sin 45° \sin 30°$

$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2}$

Simplify: $\frac{\sqrt{6} + \sqrt{2}}{4}$

Answer: $\frac{\sqrt{6} + \sqrt{2}}{4}$

4 Constant acceleration: v = u + at (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train moving in a straight line has initial speed 0 m s^{-1} and constant acceleration 0.5 m s^{-2}.
Find its speed after 5 s.

$v = u + at$

$v = 0 + 0.5 \times 5$

Answer: 2.5 m s^{-1}

5 Real roots and the discriminant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the range of values of k for which $kx^2 + 8x + 9 = 0$ has real roots.

Real roots: $b^2 - 4ac \geq 0$

$64 - 36k \geq 0$, so $k \leq \frac{16}{9}$

Also $k \neq 0$ for the equation to be a quadratic.

Answer: $k \leq \frac{16}{9}$, $k \neq 0$

6 Tree diagrams (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tree diagram shows probabilities for events R and L.
Find the probability that L happens.

P(L) = P(R and L) + P(R′ and L)

$= 0.8 \times 0.8 + 0.2 \times 0.2$

Answer: 0.68

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7 Finding a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), where f(x) is a cubic. Find f(x) in the form k(x − a)(x − b)(x − c).

The curve crosses the x-axis at x = −4, −1 and 1.
f(x) = k(x + 4)(x + 1)(x − 1)
At (0, −4): k × 4 × 1 × (−1) = −4
k = 1
Answer: f(x) = (x + 4)(x + 1)(x − 1)

8 Double angle: find sin 2A and cos 2A (All boards)

A is acute and cos A = $\frac{12}{13}$.

Find the exact values of sin 2A and cos 2A.

$\sin A = \sqrt{1 - \cos^2 A} = \frac{5}{13}$

$\sin 2A = 2 \sin A \cos A = 2 \times \frac{5}{13} \times \frac{12}{13} = \frac{120}{169}$

$\cos 2A = 2\cos^2 A - 1 = 2 \times \frac{144}{169} - 1 = \frac{119}{169}$

Answer: $\sin 2A = \frac{120}{169}$, $\cos 2A = \frac{119}{169}$

9 Velocity-time: constant deceleration to rest (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car decelerates uniformly from 10 m s^{-1} to rest in 8 s. Find (a) the deceleration, (b) the distance travelled while decelerating.

(a) $a = \frac{v - u}{t} = \frac{0 - 10}{8}$

(b) $s = \frac{(u + v)}{2} t = 5 \times 8$

Answer: (a) 1.25 m s^{-2} (b) 40 m

10 Counting: arrangements and selections (CIE)

9 runners take part in a race. In how many ways can the first 3 places be filled?

${}^9P_3 = 9 \times 8 \times 7$

Answer: 504