Day 19 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Change of sign to locate a root (All boards)

$f(x) = x^2 - \ln x - 2$.
Show that $f(x) = 0$ has a root between $x = 1$ and $x = 2$.

$f(1) = -1.00$
$f(2) = 1.31$
f is continuous and there is a change of sign, so a root lies in the interval
Answer: $f(1) = -1.00$, $f(2) = 1.31$: change of sign

2 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 5x - 2$ and $g(x) = x^2 - 3$.
Find (a) $fg(x)$, (b) $gf(x)$.

$fg(x) = f(x^2 - 3) = 5x^2 - 17$
$gf(x) = (5x - 2)^2 - 3 = 25x^2 - 20x + 1$
Answer: (a) $5x^2 - 17$ (b) $25x^2 - 20x + 1$

3 Constant acceleration: v = u + at (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car moving in a straight line has initial speed 12 m s^{-1} and constant acceleration 2.5 m s^{-2}.
Find its speed after 11 s.

$v = u + at$
$v = 12 + 2.5 \times 11$
Answer: 39.5 m s^{-1}

4 Counting: arrangements and selections (CIE)

6 runners take part in a race. In how many ways can the first 3 places be filled?

${}^6P_3 = 6 \times 5 \times 4$
Answer: 120

5 Linear and quadratic simultaneous equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations:
$y = x^2 - 7x + 4$, $y = -2x - 2$.

$x^2 - 7x + 4 = -2x - 2$
$x^2 - 5x + 6 = 0$
$(x - 3)(x - 2) = 0$
Use $y = -2x - 2$ to find each y.
Answer: $x = 2$, $y = -6$ and $x = 3$, $y = -8$

6 Binomial: finding parameters (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(n, p)$ has mean 14.4 and variance 5.76.
Find n and p.

$np = 14.4$ and $npq = 5.76$
$q = 5.76 \div 14.4 = 0.4$, so $p = 0.6$
$n = 14.4 \div 0.6$
Answer: $n = 24$, $p = 0.6$

7 Sector: arc length and area (diagram) (All boards)

The diagram shows sector OAB of a circle.
Find, in terms of π, (a) the arc length AB, (b) the area of the sector.

(a) Arc length $= r\theta = 12 \times \frac{\pi}{2} = 6\pi$ cm

(b) Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 12^2 \times \frac{\pi}{2} = 36\pi$ cm^2

Answer: (a) 6π cm (b) 36π cm^2

8 Rough slope: limiting equilibrium or sliding (All boards)

A box is on the point of sliding down a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{20}{21}$. Find the coefficient of friction.

Perpendicular: $R = mg\cos\alpha$

Parallel (limiting): $F = mg\sin\alpha$

$\mu = \frac{F}{R} = \tan\alpha$

Answer: $\boldsymbol{\frac{20}{21}}$

9 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -3x + k$ is a tangent to the curve $y = x^2 + 3x + 1$.

Find the value of k.

$x^2 + 3x + 1 = -3x + k$

$x^2 + 6x + (1 - k) = 0$

A tangent touches once: discriminant = 0

$(6)^2 - 4(1 - k) = 0$

Answer: $k = -8$

10 Separable differential equations (All boards)

Given that $dy/dx = 12x^3$ and $y = 0$ when $x = 0$, find y in terms of x.

$y = \int 12x^3\,dx = 3x^4 + c$

$x = 0, y = 0$: $c = 0$

Answer: $y = 3x^4$