Day 189 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Divide $f(x) = 2x^3 + 10x^2 + 9x - 1$ by $(x + 3)$.
Find the quotient and the remainder.

Use synthetic division with $a = -3$:
Bottom row: 2, 4, −3, 8
Quotient: $2x^2 + 4x - 3$
Remainder: 8
Answer: Quotient $2x^2 + 4x - 3$, remainder 8

2 3D vectors: distance, magnitude and midpoint (All boards)

A has coordinates (3, −2, 2) and B has coordinates (−8, −4, 12).
Find (a) the vector AB, (b) the distance AB, (c) the midpoint of AB.

(a) $\text{AB} = \text{OB} - \text{OA} = \begin{pmatrix} -11 \\ -2 \\ 10 \end{pmatrix}$
(b) $|\text{AB}| = \sqrt{11^2 + 2^2 + 10^2} = 15$
(c) Midpoint $= (-\frac{5}{2}, -3, 7)$
Answer: (a) $\begin{pmatrix} -11 \\ -2 \\ 10 \end{pmatrix}$ (b) 15 (c) $(-\frac{5}{2}, -3, 7)$

3 Arithmetic sequences: nth term (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic sequence begins 9, 15, 21, 27, ...
Find (a) an expression for the nth term,
(b) the 20th term.

$a = 9$, $d = 6$
$u_n = a + (n - 1)d = 9 + (n - 1)(6) = 6n + 3$
$u_{20} = 123$
Answer: (a) $6n + 3$ (b) 123

4 Angle between vectors (scalar product) (CIE)

$\mathbf{a} = -2\mathbf{i} - \mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$.
Find the angle between a and b, to 1 decimal place.

$\mathbf{a}.\mathbf{b} = (-2)(1) + (0)(2) + (-1)(-3) = 1$
$|\mathbf{a}| = \sqrt{5}$, $|\mathbf{b}| = \sqrt{14}$
$\cos\theta = \frac{1}{\sqrt{5} \times \sqrt{14}}$, so $\theta = 83.1°$
Answer: 83.1°

5 Lift: tension in the cable (All boards)

A lift of mass 200 kg is lowered by a vertical cable with constant acceleration 1.5 m s^{-2} downwards. Take g = 9.8 m s^{-2}.
Find the tension in the cable.

Resolving downwards: $200 \times 9.8 - T = 200 \times 1.5$
$T = 1960 - 300$
Answer: 1660 N

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6 Vector geometry with a diagram (OAB) (All boards)

In the diagram, OA = $\mathbf{a}$, OB = $\mathbf{b}$, and P lies on AB with AP : PB = 1 : 4.
Find AB and AP in terms of $\mathbf{a}$ and $\mathbf{b}$.

AB = OB − OA = $\mathbf{b} - \mathbf{a}$

AP = $\frac{1}{5}$ AB = $\frac{1}{5}(\mathbf{b} - \mathbf{a})$

Answer: AB = $\mathbf{b} - \mathbf{a}$; AP = $\frac{1}{5}(\mathbf{b} - \mathbf{a})$

7 Partial fractions (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

Express $\frac{2x + 3}{(x - 3)^2}$ in the form $\frac{A}{(x - 3)} + \frac{B}{(x - 3)^2}$.

$2x + 3 = A(x - 3) + B$
Coefficient of x: A = 2
Constant: 3 = 6 + B, so B = 9
Answer: A = 2, B = 9

8 Binomial: modelling conditions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X is the number of days in a 20 day period when it rains, where rain is more likely after a rainy day.
Can X be modelled by a binomial distribution? Give a reason.

A binomial model needs: a fixed number of trials, two outcomes, constant probability, independent trials.
Here the days are not independent.
Answer: No, the days are not independent

9 Constant acceleration: time to reach a speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A boat moving in a straight line increases its speed from 8 m s^{-1} to 20 m s^{-1} with constant acceleration 3 m s^{-2}. Find the time taken.

v = u + at
20 = 8 + 3 t
Answer: 4 s

10 Conditional probability: two-way tables (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Of 130 people, 49 are women. 23 of the women and 48 of the men walk to work.
One person is chosen at random.
Given that the person is a woman, find the probability that they walk to work.

There are 49 women
P = 23 ÷ 49
Answer: $\frac{23}{49}$