Day 188 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Straight line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find an equation of the line through A(−4, −5) and B(−5, −1), in the form ax + by + c = 0, where a, b and c are integers.

Gradient = $\frac{-1-(-5)}{-5-(-4)} = -4$

$y - (-5) = -4(x - (-4))$

Multiply out and rearrange to integer coefficients.

Answer: 4x + y + 21 = 0

2 Inverse trig functions (Edexcel, OCR)

Find the exact value, in radians, of

(a) $\arcsin(\frac{1}{2})$ (b) $\arccos(0)$

(a) $y = \arcsin(\frac{1}{2})$ means $\sin y = \frac{1}{2}$ with $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$: $y = \frac{\pi}{6}$

(b) $y = \arccos(0)$ means $\cos y = 0$ with $0 \le y \le \pi$: $y = \frac{\pi}{2}$

Answer: (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$

3 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Show, by a counter-example, that the statement "If n^2 is a multiple of 48 then n is a multiple of 48" is false, where n is a positive integer.

Look for n with n^2 divisible by 48 but n not divisible by 48.

Try n = 12: $n^2 = 144 = 48 \times 3$, a multiple of 48.

But 12 is not a multiple of 48, so the statement is false.

Answer: n = 12

4 Trapezium rule: strips shown (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows the curve $y = e^{\frac{x}{2}}$ from x = 0 to x = 1 divided into 6 strips of equal width.
Use the trapezium rule with these strips to estimate the shaded area to 3 s.f.

h = 0.16666666666666666; y values: 1.000, 1.087, 1.181, 1.284, 1.396, 1.517, 1.649

Area $\approx \frac{0.16666666666666666}{2}[1.000 + 1.649 + 2(6.465)]$

= 1.30

Answer: 1.30

5 Inclined plane: normal reaction and holding force (All boards)

A particle of mass 5 kg rests on a smooth plane inclined at α to the horizontal, where $\tan\alpha = \frac{3}{4}$.

It is held by a force P N up the slope. Take g = 9.8 m s^{-2}. Find P and the normal reaction.

Parallel: $P = mg\sin\alpha = 5 \times 9.8 \times \frac{3}{5}$

Perpendicular: $R = mg\cos\alpha = 5 \times 9.8 \times \frac{4}{5}$

Answer: P = 29.4, R = 39.2

6 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = x^2 + 2x$ is translated by the vector $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

Find the equation of the new curve, giving your answer in the form $y = ax^2 + bx + c$.

Replace x by (x − 2) and add −1:

$y = f(x - 2) - 1$

Expand and collect terms.

Answer: $y = x^2 - 2x - 1$

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7 Resultant force as vectors and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(8\mathbf{i})$ N and $(-2\mathbf{i} + 3\mathbf{j})$ N act on a particle of mass 3 kg. No other horizontal forces act. Find the acceleration of the particle as a vector in terms of $\mathbf{i}$ and $\mathbf{j}$.

Resultant = $(8\mathbf{i}) + (-2\mathbf{i} + 3\mathbf{j})$
= $(6\mathbf{i} + 3\mathbf{j})$ N

F = ma: $a = \frac{1}{3}(6\mathbf{i} + 3\mathbf{j})$

Answer: $(2\mathbf{i} + \mathbf{j})$ m s^{-2}

8 Two-tailed binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(12, p). H_0: p = 0.4, H_1: p ≠ 0.4. In a sample, x = 9.
Test at the 10% significance level.

Two-tailed test, so compare with 0.1 ÷ 2 = 0.05
P(X ≥ 9) = 0.0153
0.0153 < 0.05, so reject H_0

Answer: Reject H_0 (P(X ≥ 9) = 0.0153)

9 Area enclosed by a parabola (diagram) (All boards)

The diagram shows the curve y = x(8 − x).
Find the area of the shaded region, between the curve and the x-axis.

The curve meets the x-axis where x = 0 and x = 8

Area = $\int_0^8 8x - x^2\,dx = [\frac{8x^2}{2} - \frac{x^3}{3}]$ from 0 to 8

$= \frac{8^3}{2} - \frac{8^3}{3}$

$= \frac{256}{3}$

Answer: $\frac{256}{3}$

10 Sampling: advantages (AQA, Edexcel, OCR, MEI, Eduqas)

A researcher wants to find out the satisfaction of the customers of a shop.
Give one advantage and one disadvantage of using systematic sampling.

Advantage: it is quick and easy and spreads the sample evenly through the list.
Disadvantage: a repeating pattern in the list could cause bias, and a list is needed.

Answer: Advantage: quick and evenly spread. Disadvantage: a pattern in the list may bias