Day 187 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Straight line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find an equation of the line through A(−6, 7) and B(−1, 1), in the form ax + by + c = 0, where a, b and c are integers.

Gradient = $\frac{1-7}{-1-(-6)} = -\frac{6}{5}$

$y - 7 = -\frac{6}{5}(x - (-6))$

Multiply out and rearrange to integer coefficients.

Answer: 6x + 5y + 1 = 0

2 Constant acceleration: time to reach a speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist moving in a straight line increases its speed from 7 m s^{-1} to 12 m s^{-1} with constant acceleration 0.5 m s^{-2}. Find the time taken.

$v = u + at$

$12 = 7 + 0.5\,t$

Answer: 10 s

3 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $(n + 2)^2 - (n - 2)^2$ is a multiple of 4 for every integer n.

$(n + 2)^2 - (n - 2)^2 = (n^2 + 4n + 4) - (n^2 - 4n + 4)$

$= 8n = 4 \times 2n$

2n is an integer, so the expression is a multiple of 4.

Answer: Shown

4 Median and interquartile range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Here are 15 values: 72, 61, 69, 45, 79, 29, 77, 63, 57, 75, 76, 7, 42, 6, 23

Find the median and the interquartile range.

Ordered: 6, 7, 23, 29, 42, 45, 57, 61, 63, 69, 72, 75, 76, 77, 79

Median = 61, lower quartile = 29, upper quartile = 75

IQR = 75 − 29

Answer: median = 61, IQR = 46

5 Arc length and perimeter of a sector (All boards)

A sector of a circle has radius 5 cm and angle 1.6 radians.

Find (a) the arc length, (b) the perimeter of the sector.

(a) Arc length = $r\theta = 5 \times 1.6 = 8.00$ cm

(b) Perimeter = arc + 2r = 8.00 + 10 = 18.0 cm

Answer: (a) 8.00 cm (b) 18.0 cm

6 Projectile launched with velocity components (AQA, Edexcel, OCR, MEI)

A ball is projected from level ground with velocity (4**i** + 19.6**j**) m s^{-1} (**i** horizontal, **j** vertically up). Take g = 9.8 m s^{-2}.

(a) Find the time of flight.

(b) Find the horizontal range.

(a) Vertically: $0 = 19.6\,t - 4.9t^2$

$t = \frac{19.6}{4.9} = 4$

(b) Range = 4 × 4

Answer: (a) 4 s (b) 16 m

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7 Angle between vectors (scalar product) (CIE)

$\mathbf{a} = -3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + 4\mathbf{j} + \mathbf{k}$.
Find the angle between a and b, to 1 decimal place.

$\mathbf{a}.\mathbf{b} = (-3)(-2) + (-1)(4) + (4)(1) = 6$

$|\mathbf{a}| = \sqrt{26}$, $|\mathbf{b}| = \sqrt{21}$

$\cos\theta = \frac{6}{\sqrt{26} \times \sqrt{21}}$, so $\theta = 75.1°$

Answer: 75.1°

8 Equation of a reciprocal graph from asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the graph of $y = \frac{k}{x - a} + c$ with its asymptotes. Find the values of a, c and k.

The asymptotes are $x = 2$ and $y = 2$, so $a = 2$ and $c = 2$.

Through (0, 4): $4 = \frac{k}{0 - 2} + 2$

$\frac{k}{-2} = 2$, so $k = -4$

Answer: $a = 2$, $c = 2$, $k = -4$

9 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a machine is set up wrongly is 0.3. If so, the probability that an item is faulty is 0.8; otherwise it is 0.1. Given that an item is faulty, find the probability that a machine is set up wrongly.

P(both) = 0.3 × 0.8 = 0.24; P(not first and second) = 0.7 × 0.1 = 0.07

P(second) = 0.24 + 0.07 = 0.31

P = 0.24 ÷ 0.31

Answer: $\frac{24}{31} = 0.774$

10 Vector geometry with a diagram (OAB) (All boards)

In the diagram, OA = $\mathbf{a}$, OB = $\mathbf{b}$, and P lies on AB with AP : PB = 4 : 3.
Find AB and OP in terms of $\mathbf{a}$ and $\mathbf{b}$.

$\mathrm{AB} = \mathrm{OB} - \mathrm{OA} = \mathbf{b} - \mathbf{a}$

$\mathrm{AP} = \frac{4}{7}\mathrm{AB} = \frac{4}{7}(\mathbf{b} - \mathbf{a})$

$\mathrm{OP} = \mathrm{OA} + \mathrm{AP} = \mathbf{a} + \frac{4}{7}(\mathbf{b} - \mathbf{a})$

$= \frac{3}{7}\mathbf{a} + \frac{4}{7}\mathbf{b}$

Answer: $\mathbf{AB} = \mathbf{b} - \mathbf{a}$; $\mathbf{OP} = \frac{3}{7}\mathbf{a} + \frac{4}{7}\mathbf{b}$