Day 186 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Position vector under constant velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A drone moves with constant velocity $(-6\mathbf{i} + 8\mathbf{j})$ m s^{-1}.
At time t = 0 its position vector is $(-4\mathbf{i} - \mathbf{j})$ m.
(a) Find its position vector when t = 6.
(b) Find the distance it travels in 6 s.

(a) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = (-4\mathbf{i} - \mathbf{j}) + 6(-6\mathbf{i} + 8\mathbf{j})$

(b) speed $= \sqrt{6^2 + 8^2} = 10$ m s^{-1}
distance $= 10 \times 6$
Answer: (a) $(-40\mathbf{i} + 47\mathbf{j})$ m (b) 60 m

2 Natural logs and e^x (All boards)

Solve $e^{2x+3} = 50$.
Give your answer to 3 significant figures.

Take natural logs: $2x + 3 = \ln 50 = 3.912$
$x = 0.456$
Answer: x = 0.456

3 Line through two axis intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a line meeting the axes at A and B.
Find (a) the equation of AB in the form px + qy = r with integers p, q and r, (b) the length AB.

Gradient $= \frac{-4 - 0}{0 - 2} = 2$
Using y = mx + c with c = −4: y = 2x − 4, so 2x − y = 4
$AB^2 = 2^2 + 4^2 = 20$
Answer: (a) 2x − y = 4 (b) $2\sqrt{5}$

4 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 5^2)$. H_0: $\mu = 35$, H_1: $\mu < 35$.
A sample of 9 has mean $\bar{x} = 31$.
Test at the 5% level. Critical value: z = −1.645.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (31 - 35) \div (5 \div 3)$
$z = -2.40$
−2.40 is inside the critical region, so reject H_0
Answer: z = −2.40; reject H_0

5 Solve a logarithmic equation (two logs) (All boards)

Solve $\ln(x + 8) - \ln x = \ln 5$.

$\ln \frac{x+8}{x} = \ln 5$, so $\frac{x+8}{x} = 5$
$x + 8 = 5x$, so $8 = 4x$
$x = 2$
Answer: x = 2

6 Linear and quadratic simultaneous equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations:
$y = x^2 - 3x + 1$, $y = 3x - 4$.

$x^2 - 3x + 1 = 3x - 4$
$x^2 - 6x + 5 = 0$
$(x - 1)(x - 5) = 0$
Use $y = 3x - 4$ to find each y.
Answer: x = 1, y = −1 and x = 5, y = 11

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7 Angle between vectors (scalar product) (CIE)

$\mathbf{a} = 3\mathbf{j} - \mathbf{k}$ and $\mathbf{b} = \mathbf{i} - \mathbf{j} - \mathbf{k}$.

Find the angle between a and b, to 1 decimal place.

$\mathbf{a}.\mathbf{b} = (0)(1) + (3)(-1) + (-1)(-1) = -2$

$|\mathbf{a}| = \sqrt{10}$, $|\mathbf{b}| = \sqrt{3}$

$\cos\theta = \frac{-2}{\sqrt{10} \times \sqrt{3}}$, so $\theta = 111.4°$

Answer: 111.4°

8 Differentiating displacement to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its displacement from O at time t seconds is s metres, where $s = 3t^3 + 3t^2 + 4t$.

Find its velocity when t = 2.

$v = \frac{ds}{dt} = 9t^2 + 6t + 4$

When t = 2: $v = 9 \times 2^2 + 6 \times 2 + 4$

Answer: 52 m s^{-1}

9 Coding (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The values x are coded using $y = (x - 50) \div 5$.

The coded values have mean $\bar{y} = 2.9$ and standard deviation 0.5.

Find the mean and standard deviation of x.

$x = 50 + 5y$

Mean: $50 + 5 \times 2.9 = 64.5$

Standard deviation: $5 \times 0.5 = 2.5$ (adding 50 has no effect)

Answer: mean = 64.5, standard deviation = 2.5

10 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = x^2 + 2x - 1$ is translated by the vector $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$.

Find the equation of the new curve, giving your answer in the form $y = ax^2 + bx + c$.

Replace x by (x + 2) and add 4:

$y = f(x + 2) + 4$

Expand and collect terms.

Answer: $y = x^2 + 6x + 11$