Day 185 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 F = ma: find the acceleration (All boards)

A horizontal force of 45 N acts on a sledge of mass 10 kg on a smooth horizontal surface.
Find the acceleration.

$F = ma$
$45 = 10\ a$
Answer: 4.5 m s^{-2}

2 Midpoint and length of a line segment (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is $(-4, 5)$ and B is $(2, -1)$.
Find (a) the midpoint of AB,
(b) the length of AB, as a surd in simplest form.

(a) Midpoint $= \left(\frac{-4+2}{2}, \frac{5+(-1)}{2}\right) = (-1, 2)$

(b) $AB^2 = (6)^2 + (-6)^2 = 36 + 36 = 72$
$AB = \sqrt{72} = 6\sqrt{2}$
Answer: (a) $(-1, 2)$ (b) $6\sqrt{2}$

3 Change of sign to locate a root (All boards)

$f(x) = e^x - 3x - 2$.
Show that $f(x) = 0$ has a root between $x = 2$ and $x = 3$.

$f(2) = -0.611$
$f(3) = 9.09$
f is continuous and there is a change of sign, so a root lies in the interval
Answer: f(2) = −0.611, f(3) = 9.09: change of sign

4 Binomial: modelling conditions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X is the number of heads when a fair coin is tossed 7 times.
Can X be modelled by a binomial distribution? Give a reason.

A binomial model needs: a fixed number of trials, two outcomes, constant probability, independent trials.
Here a fixed number of independent trials, each with probability 0.5 of a head.
Answer: Yes, a fixed number of independent trials, each with probability 0.5 of a head

5 Position of a point relative to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $(x + 3)^2 + (y + 4)^2 = 25$.
Determine whether the point $P(6, -2)$ is inside, outside or on the circle.

Distance^2 from the centre $(-3, -4)$ to P:
$(6 - (-3))^2 + (-2 - (-4))^2 = 81 + 4 = 85$
$r^2 = 25$, and $85 > 25$.
Answer: P is outside the circle

6 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(30, p)$. H_0: $p = 0.2$, H_1: $p < 0.2$.
Find the critical region for a test at the 10% significance level.

$P(X \leq 2) = 0.0442 \leq 0.1$
$P(X \leq 3) = 0.123 > 0.1$
Answer: $X \leq 2$

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7 Two-stage motion: acceleration then constant speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car starts from rest and accelerates uniformly at 0.5 m s^{-2} for 8 s, then travels at constant speed for a further 20 s. Find the total distance travelled.

Speed reached: $v = 0.5 \times 8 = 4$ m s^{-1}

Stage 1: $s = \frac{1}{2} \times 4 \times 8 = 16$

Stage 2: $s = 4 \times 20 = 80$

Answer: 96 m

8 Integration by parts (All boards)

Use integration by parts to find
$\int x \cos 5x \, dx$.

$u = x$, $dv/dx = \cos 5x$; $du/dx = 1$, $v = \frac{\sin 5x}{5}$

$\int u \, (dv/dx) \, dx = uv - \int v \, (du/dx) \, dx$

Evaluate the remaining integral and simplify

Answer: $\frac{x \sin 5x}{5} + \frac{\cos 5x}{25} + c$

9 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -4x + k$ is a tangent to the curve
$y = x^2 - 10x + 5$.
Find the value of k.

$x^2 - 10x + 5 = -4x + k$

$x^2 - 6x + (5 - k) = 0$

A tangent touches once: discriminant = 0

$(-6)^2 - 4(5 - k) = 0$

Answer: $k = -4$

10 Collinear points using vectors (All boards)

A(0, −2), B(4, −4) and C(10, k).
Given that A, B and C are collinear, find k.

$\mathrm{AB} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$, $\mathrm{AC} = \begin{pmatrix} 10 \\ k + 2 \end{pmatrix}$

$\mathrm{AC} = \frac{5}{2}\,\mathrm{AB}$, so $10 = \frac{5}{2} \times 4$, giving the multiple $\frac{5}{2}$

$k + 2 = \frac{5}{2} \times (-2)$, so $k = -7$

Answer: $k = -7$