Day 184 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $3x^2 + 18x + 33$ in the form $a(x + p)^2 + q$.

Take out a factor 3: $3(x^2 + 6x + 11)$

$x + 6 = (x + 3)^2 - 9$

$3[(x + 3)^2 - 9] + 33 = 3(x + 3)^2 + 6$

Answer: $3(x + 3)^2 + 6$

2 3D vectors: distance, magnitude and midpoint (All boards)

A has coordinates (0, 3, 2) and B has coordinates (−2, 0, 8).
Find (a) the vector AB, (b) the distance AB, (c) the midpoint of AB.

(a) $\text{AB} = \text{OB} - \text{OA} = \begin{pmatrix} -2 \\ -3 \\ 6 \end{pmatrix}$

(b) $|\text{AB}| = \sqrt{2^2 + 3^2 + 6^2} = 7$

(c) Midpoint $= (-1, \frac{3}{2}, 5)$

Answer: (a) $\begin{pmatrix} -2 \\ -3 \\ 6 \end{pmatrix}$ (b) 7 (c) $(-1, \frac{3}{2}, 5)$

3 Selection without replacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A bag has 6 red and 4 blue sweets. Two are taken at random without replacement.
Find the probability that at least one of them is red.

$\text{P(no red)} = \text{P(BB)} = \frac{4}{10} \times \frac{3}{9} = \frac{2}{15}$

$\text{P(at least one red)} = 1 - \frac{2}{15}$

Answer: $\frac{13}{15}$

4 Average speed over two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A boat travels 9 m at a constant speed of 3 m s^{-1}, then a further 14 m at a constant speed of 2 m s^{-1}. Find the average speed for the whole journey.

Time for first stage = 9 ÷ 3 = 3 s

Time for second stage = 14 ÷ 2 = 7 s

$\text{Average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{23}{10}$

Answer: 2.3 m s^{-1}

5 Angle between vectors (scalar product) (CIE)

$\mathbf{a} = \mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$.
Find the angle between a and b, to 1 decimal place.

$\mathbf{a}.\mathbf{b} = (1)(-2) + (1)(-2) + (-1)(-3) = -1$

$|\mathbf{a}| = \sqrt{3}$, $|\mathbf{b}| = \sqrt{17}$

$\cos\theta = \frac{-1}{\sqrt{3} \times \sqrt{17}}$, so $\theta = 98.0°$

Answer: 98.0°

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6 Reciprocal graphs and asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = \frac{-2}{x-1} - 3$.

Write down the equations of the asymptotes and the coordinates where the curve meets the axes.

Asymptotes: $x = 1$ and $y = -3$.

$y = 0$: $\frac{-2}{x-1} = 3$, so $x = \frac{1}{3}$

$x = 0$: $y = \frac{-2}{-1} - 3 = -1$

Answer: $x = 1$, $y = -3$; $(\frac{1}{3}, 0)$, $(0, -1)$

7 Normal distribution: probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(55, 2^2)$. Find $P(X > 57)$, giving your answer to 3 s.f.

$z = (57 - 55) \div 2 = 1$

$P(X > 57) = P(Z > 1)$

Answer: 0.159

8 Constant velocity vector: when does it reach a point? (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves with constant velocity

$(5\mathbf{i} + 2\mathbf{j})$ m s^{-1}. It is at the point with position vector $(-2\mathbf{j})$ m at $t = 0$. Find the time when its position vector is $(20\mathbf{i} + 6\mathbf{j})$ m.

$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$

$(20\mathbf{i} + 6\mathbf{j}) = (-2\mathbf{j}) + t(5\mathbf{i} + 2\mathbf{j})$

Equating i components: $20 = 0 + 5t$

Answer: 4 s

9 Factorised trig equation (All boards)

Solve $\cos x(2\cos x - 1) = 0$ for $0° \leq x \leq 360°$.

Either $2\cos x - 1 = 0$ or $\cos x = 0$

$\cos x = \frac{1}{2}$: $x = 60°, 300°$

$\cos x = 0$: $x = 90°, 270°$

Answer: $x = 60°, 90°, 270°, 300°$

10 Combining two inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x satisfying both

$3x + 1 < 1$ and $x^2 - 3x - 10 < 0$.

$3x + 1 < 1$ gives $x < 0$.

$x^2 - 3x - 10 < 0$: $(x + 2)(x - 5) < 0$, so $-2 < x < 5$.

Both are true where the two ranges overlap.

Answer: $-2 < x < 0$