Day 183 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\sqrt{3} + \sqrt{108}$, giving your answer in the form $p\sqrt{3}$.

$\sqrt{3} = 1\sqrt{3}$ and $\sqrt{108} = 6\sqrt{3}$

Sum = $(1 + 6)\sqrt{3}$

Answer: $7\sqrt{3}$

2 Stationary points of a cubic (All boards)

$y = x^3 + 6x^2 + 9x - 1$.

Find the coordinates of the stationary points and determine their nature.

$dy/dx = 3x^2 + 12x + 9 = 3(x + 3)(x + 1) = 0$

$x = -3$: $y = -1$; $x = -1$: $y = -5$

$d^2y/dx^2 = 6x + 12$: at $x = -3$ it is -6 (maximum), at $x = -1$ it is 6 (minimum)

Answer: $(-3, -1)$ maximum; $(-1, -5)$ minimum

3 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $3x^2 - 6x - 4$ in the form $a(x + p)^2 + q$.

Take out a factor 3: $3(x^2 - 2x - \frac{4}{3})$

$x - 2 = (x - 1)^2 - 1$

$3[(x - 1)^2 - 1] - 4 = 3(x - 1)^2 - 7$

Answer: $3(x - 1)^2 - 7$

4 Definite integrals (exact) (All boards)

Find the exact value of

$\int_1^3 \frac{5}{x}\,dx$.

[5 ln x] from 1 to 3

$= 5 \ln 3 - 5 \ln 1 = 5 \ln 3$

Answer: 5 ln 3

5 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

30 pairs of data on height and arm span have $r = -0.26$.

Test, at the 5% significance level, for evidence of negative correlation. The critical value is -0.3061.

H_0: $\rho = 0$, H_1: $\rho < 0$

$r = -0.26$ is not beyond the critical value -0.3061

Do not reject H_0

Answer: Do not reject H_0: there is not enough evidence of negative correlation

6 Catching up: constant speed against acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes a point O at constant speed 24 m s^{-1}.

At that instant car B starts from rest at O with constant acceleration 4 m s^{-2} in the same direction. Find the time when B catches up A.

Distances equal: $24\,t = \frac{1}{2} \times 4 \times t^2$

$t(24 - 2\,t) = 0$, so $t = 0$ or $t = 24 \div 2$

Answer: 12 s

7 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(40, p)$. H_0: $p = 0.2$, H_1: $p > 0.2$.

Find the critical region for a test at the 1% significance level.

$P(X \geq 15) = 0.00792 \leq 0.01$

$P(X \geq 14) = 0.0194 > 0.01$

Answer: $X \geq 15$

Day 183 **A Level Maths: C Tier 10-a-day: Model Solutions**

8 Car towing a caravan (All boards)

A car of mass 1500 kg tows a caravan of mass 400 kg on a level road with acceleration 1 m s^{-2}. Resistances: 300 N on the car, 100 N on the caravan. Find the driving force and the tow bar tension.

Whole system: $D - 300 - 100 = 1900 \times 1$

Caravan: $T - 100 = 400 \times 1$

Answer: D = 2300 N, T = 500 N

9 Angle between vectors (scalar product) (CIE)

$\mathbf{a} = \mathbf{i} - \mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + 2\mathbf{j}$.

Find the angle between a and b, to 1 decimal place.

$\mathbf{a}.\mathbf{b} = (1)(2) + (-1)(2) + (-3)(0) = 0$

$|\mathbf{a}| = \sqrt{11}$, $|\mathbf{b}| = \sqrt{8}$

$\cos\theta = \frac{0}{\sqrt{11} \times \sqrt{8}}$, so $\theta = 90.0°$

Answer: 90.0°

10 Equation of a reciprocal graph from asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the graph of $y = \frac{k}{x - a} + c$ with its asymptotes. Find the values of a, c and k.

The asymptotes are $x = 1$ and $y = -2$, so $a = 1$ and $c = -2$.

Through (0, 1): $1 = \frac{k}{0 - 1} + -2$

$\frac{k}{-1} = 3$, so $k = -3$

Answer: a = 1, c = −2, k = −3