Day 182 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Types of data (AQA, Edexcel, OCR, MEI, Eduqas)

Classify each variable as qualitative, discrete or continuous.
(a) the number of cars in a car park
(b) the time taken to run 100 m
(c) the number of siblings a student has

(a) discrete: it is counted, so takes separate values
(b) continuous: it is measured, so can take any value in a range
(c) discrete: it is counted, so takes separate values
Answer: (a) discrete; (b) continuous; (c) discrete

2 3D vectors: distance, magnitude and midpoint (All boards)

$\text{OA} = \begin{pmatrix} 5 \\ -1 \\ -3 \end{pmatrix}$ and $\text{OB} = \begin{pmatrix} 9 \\ 3 \\ 4 \end{pmatrix}$.
Find AB and |AB|, and a unit vector in the direction of AB.

$\text{AB} = \text{OB} - \text{OA} = \begin{pmatrix} 4 \\ 4 \\ 7 \end{pmatrix}$
$|\text{AB}| = \sqrt{4^2 + 4^2 + 7^2} = 9$
Unit vector $= \frac{1}{9}$ AB
Answer: AB $= \begin{pmatrix} 4 \\ 4 \\ 7 \end{pmatrix}$**, |AB| = 9, unit vector** $\frac{1}{9}\begin{pmatrix} 4 \\ 4 \\ 7 \end{pmatrix}$

3 Constant acceleration: $v^2 = u^2 + 2as$ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line with constant acceleration. Its speed increases from 7 m s^{-1} to 11 m s^{-1} as it travels 24 m. Find the acceleration.

$v^2 = u^2 + 2as$
$11^2 = 7^2 + 2 \times a \times 24$
$72 = 48\,a$
Answer: 1.5 m $\mathbf{s^{-2}}$

4 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = x^2 - x + 3$ is translated by the vector $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$.
Find the equation of the new curve, giving your answer in the form $y = ax^2 + bx + c$.

Replace x by $(x - 2)$ and add 2:
$y = f(x - 2) + 2$
Expand and collect terms.
Answer: $\mathbf{y = x^2 - 5x + 11}$

5 Horizontal projection from a cliff (AQA, Edexcel, OCR, MEI)

A ball is thrown horizontally at 10 m s^{-1} from the top of a vertical cliff of height 78.4 m above the sea. Take g = 9.8 m s^{-2}. Find (a) the time taken to reach the sea, (b) the horizontal distance of the landing point from the cliff.

(a) Vertically: u = 0, s = 78.4, a = 9.8
$78.4 = \frac{1}{2} \times 9.8 \times t^2$, so $t^2 = 16$
(b) Horizontal distance = speed × time = 10 × 4
Answer: (a) 4 s (b) 40 m

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6 Maximum value from R form (All boards)

f(x) = 4 sin x + 3 cos x + 1.
Write f(x) in the form R sin(x + α) + 1, with α in degrees to 1 d.p., and state the maximum value of f(x).

$R = \sqrt{4^2 + 3^2} = 5$, $\tan \alpha = \frac{3}{4}$ so $\alpha = 36.9°$

f(x) = 5 sin(x + 36.9°) + 1
sin(x + α) has maximum value 1, so f(max) = 5 + 1
Answer: f(x) = 5 sin(x + 36.9°) + 1; maximum 6

7 Finding a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), where f(x) is a cubic. Find f(x) in the form k(x − a)(x − b)(x − c).

The curve crosses the x-axis at x = −2, 2 and 4.
f(x) = k(x + 2)(x − 2)(x − 4)
At (0, −32): k × 2 × (−2) × (−4) = −32
k = −2
Answer: f(x) = −2(x + 2)(x − 2)(x − 4)

8 Separable differential equations (All boards)

Given that $dy/dx = 5x^2$ and y = 3 when x = 0,
find y in terms of x.

$y = \int 5x^2 \, dx = \frac{5x^3}{3} + c$

x = 0, y = 3: c = 3

Answer: $y = \frac{5x^3}{3} + 3$

9 Frequency density (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In a histogram, the class 40 to 50 has frequency 30 and is drawn with a bar of height 6 cm.
The class 50 to 70 has frequency 30. Find the height of its bar.

Frequency density = frequency ÷ class width
First bar: 30 ÷ 10 = 3, so 6 cm represents 3
Second bar: 30 ÷ 20 = 1.5

Height = $6 \times \frac{1.5}{3}$

Answer: 3 cm

10 Straight line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find an equation of the line through A(7, 0) and B(2, −1), in the form ax + by + c = 0, where a, b and c are integers.

Gradient = $\frac{-1 - 0}{2 - 7} = \frac{1}{5}$

$y - 0 = \frac{1}{5}(x - 7)$

Multiply out and rearrange to integer coefficients.
Answer: x − 5y − 7 = 0