Day 180 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2x - 7 < -4x + 6$.

Collect x terms: $6x < 13$

Divide by 6: $x < \frac{13}{6}$

Answer: $x < \frac{13}{6}$

2 Area enclosed by a parabola (diagram) (All boards)

The diagram shows the curve $y = 3x(8 - x)$.
Find the area of the shaded region, between the curve and the x-axis.

The curve meets the x-axis where $x = 0$ and $x = 8$

Area $= \int_0^8 3(8x - x^2)\,dx = [3(\frac{8x^2}{2} - \frac{x^3}{3})]$ from 0 to 8

$= 3(\frac{8^3}{2} - \frac{8^3}{3})$

$= 256$

Answer: 256

3 Line through two axis intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a line meeting the axes at A and B.
Find (a) the equation of AB in the form $px + qy = r$ with integers p, q and r, (b) the length AB.

Gradient $= \frac{6 - 0}{0 - 4} = -\frac{3}{2}$

Using $y = mx + c$ with $c = 6$: $y = -\frac{3}{2}x + 6$, so $3x + 2y = 12$

$AB^2 = 4^2 + 6^2 = 52$

Answer: (a) $3x + 2y = 12$ (b) $2\sqrt{13}$

4 Counting: arrangements and selections (CIE)

How many different arrangements are there of the letters of the word COFFEE?

6 letters, with repeats: F appears 2 times, E appears 2 times

$6! \div (2! \times 2!)$

Answer: 180

5 Geometric series: sum of n terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 5 terms of the geometric series $6 + 18 + 54 + ...$

$a = 6$, $r = 3$, $n = 5$

$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{6(3^5 - 1)}{3 - 1}$

$= 726$

Answer: 726

6 Friction: acceleration along rough ground (All boards)

A block of mass 5 kg is pulled along rough horizontal ground by a horizontal force of 27 N.
The coefficient of friction is 0.5. Take $g = 9.8$ m s^{-2}. Find the acceleration.

$R = 5 \times 9.8 = 49$

$F = \mu R = 0.5 \times 49 = 24.5$

$P - F = ma$: $27 - 24.5 = 5a$

Answer: 0.5 m s^{-2}

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7 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $y^2 = x + 8$.
Find dy/dx in terms of x and y, and hence the gradient at (1, 3).

Differentiate both sides with respect to x (using dy/dx for y terms)

$\frac{dy}{dx} = \frac{1}{2y}$

At (1, 3): $\frac{dy}{dx} = \frac{1}{6}$

Answer: $\frac{dy}{dx} = \frac{1}{2y}$; gradient $\frac{1}{6}$

8 Projectile launched with velocity components (AQA, Edexcel, OCR, MEI)

A ball is projected from level ground with velocity $(12\mathbf{i} + 19.6\mathbf{j})$ m s^{-1} ($\mathbf{i}$ horizontal, $\mathbf{j}$ vertically up). Take g = 9.8 m s^{-2}.
(a) Find the time of flight.
(b) Find the horizontal range.

(a) Vertically: $0 = 19.6\,t - 4.9t^2$

$t = \frac{19.6}{4.9} = 4$

(b) Range = 12×4

Answer: (a) 4 s (b) 48 m

9 Grouped data: estimated mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows grouped data.
Class: $20 \le x < 25$; $25 \le x < 30$; $30 \le x < 35$; $35 \le x < 40$
Frequency: 7, 9, 4, 12
Estimate the mean, to 3 s.f.

Midpoints: 22.5, 27.5, 32.5, 37.5
$\Sigma fx = 7\times22.5 + 9\times27.5 + 4\times32.5 + 12\times37.5 = 985$
Mean $\approx 985 \div 32$
Answer: 30.8

10 Integrate using partial fractions (All boards)

$\frac{5x + 11}{(x + 1)(x + 3)} \equiv \frac{A}{x + 1} + \frac{B}{x + 3}$.

Find A and B, and hence find $\int \frac{5x + 11}{(x + 1)(x + 3)}\,dx$.

$5x + 11 \equiv A(x + 3) + B(x + 1)$
$x = -1$ gives $A = 3$; $x = -3$ gives $B = 2$
$\int \left(\frac{3}{x + 1} + \frac{2}{x + 3}\right) dx = 3 \ln|x + 1| + 2 \ln|x + 3| + c$
Answer: A = 3, B = 2; $3 \ln|x + 1| + 2 \ln|x + 3| + c$