Day 18 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Constant acceleration: distance from average speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car moving in a straight line with constant acceleration increases its speed from 4 m s^{-1} to 14 m s^{-1} in 7 s.
Find the distance it travels in this time.

$s = \frac{(u + v)}{2} t$

$s = \frac{(4 + 14)}{2} \times 7$

Answer: 63 m

2 Hypotheses for a binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A company says that the probability a customer buys a ticket is 0.75.
A researcher thinks that the probability is different. Let p be the probability that customer buys a ticket.
Write down suitable hypotheses for a test on p.

H_0 states the claimed value; H_1 matches the researcher's belief.

Two-tailed test

Answer: H_0: p = 0.75, H_1: p ≠ 0.75

3 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (−6, −5) and B is (−12, −13).
Find the equation of the perpendicular bisector of AB, in the form ax + by = c with integer a, b and c.

Midpoint of AB = (−9, −9)

Gradient of AB = $\frac{-8}{-6} = \frac{4}{3}$, so perpendicular gradient = $-\frac{3}{4}$

$y - (-9) = -\frac{3}{4}(x - (-9))$

Multiply out and rearrange.

Answer: 3x + 4y = −63

4 Inverse trig functions (Edexcel, OCR)

Find the exact value, in radians, of
(a) arccos(−1) (b) arccos(1)

(a) y = arccos(−1) means cos y = −1 with $0 \le y \le \pi$: $y = \pi$
(b) y = arccos(1) means cos y = 1 with $0 \le y \le \pi$: y = 0

Answer: (a) π (b) 0

5 Equation of a circle from a diagram (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a circle with the centre marked.
The circle touches the x-axis. Find the equation of the circle.

The circle touches the x-axis, so the radius equals the y-coordinate of the centre: r = 2.

Centre (2, 2): $(x - a)^2 + (y - b)^2 = r^2$

Answer: $(x - 2)^2 + (y - 2)^2 = 4$

6 Solve a logarithmic equation (two logs) (All boards)

Solve ln (x + 4) − ln x = ln 5.

$\ln \frac{x + 4}{x} = \ln 5$, so $\frac{x + 4}{x} = 5$

x + 4 = 5x, so 4 = 4x

x = 1

Answer: x = 1

7 Moments: reactions on a uniform beam (AQA, Edexcel, OCR, MEI)

A uniform beam AB of mass 15 kg and length 8 m rests horizontally on supports at A and B. A load of mass 25 kg is placed on the beam 2 m from A. Take g = 9.8 m s^{-2}. Find the reaction at B.

Moments about A:

$R_B \times 8 = 15 \times 9.8 \times 4 + 25 \times 9.8 \times 2$

$R_B \times 8 = 1078$

Answer: 134.75 N

8 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a machine is set up wrongly is 0.5. If so, the probability that an item is faulty is 0.6; otherwise it is 0.1. Given that an item is faulty, find the probability that a machine is set up wrongly.

P(both) = 0.5 × 0.6 = 0.3; P(not first and second) = 0.5 × 0.1 = 0.05

P(second) = 0.3 + 0.05 = 0.35

P = 0.3 ÷ 0.35

Answer: $\frac{6}{7}$ = 0.857

9 Double angle equation (All boards)

Solve $\cos 2x = -\frac{\sqrt{3}}{2}$ for $0° \leq x \leq 180°$.

Let u = 2x, so $0° \leq u \leq 360°$

$\cos u = -\frac{\sqrt{3}}{2}$: u = 150°, 210°, 510°, 570°

Divide by 2: x = 75°, 105°

Answer: x = 75°, 105°

10 Discriminant: equal roots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$x^2 + kx + 9 = 0$ has equal roots.

Find the possible values of k.

Equal roots: $b^2 - 4ac = 0$

$k^2 - 4 \times 1 \times 9 = 0$, so $k^2 = 36$

Answer: k = ±6