Day 179 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Finding a quadratic from its turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of a quadratic curve.

Find its equation in the form $y = a(x - p)^2 + q$.

The turning point is (3, 5), so $y = a(x - 3)^2 + 5$.

The curve passes through (0, −4):

$-4 = a(-3)^2 + 5$, so $-9 = 9a$

$a = -1$

Answer: $y = -(x - 3)^2 + 5$

2 Normal distribution: diagram and symmetry (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the density of $X \sim N(125, 3^2)$.

a and b are equal distances from the mean. The shaded area is 0.49.

Find $P(X > b)$.

Unshaded area = 1 − 0.49 = 0.51

The two tails are equal by symmetry, so $P(X > b) = 0.51 \div 2$

Answer: 0.255

3 Double angle: find sin 2A and cos 2A (All boards)

A is acute and $\sin A = \frac{4}{5}$.

Find the exact values of sin 2A and cos 2A.

$\cos A = \sqrt{1 - \sin^2 A} = \frac{3}{5}$

$\sin 2A = 2 \sin A \cos A = 2 \times \frac{4}{5} \times \frac{3}{5} = \frac{24}{25}$

$\cos 2A = \cos^2 A - \sin^2 A = \frac{9}{25} - \frac{16}{25} = -\frac{7}{25}$

Answer: $\sin 2A = \frac{24}{25}$, $\cos 2A = -\frac{7}{25}$

4 Differentiating velocity to find acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds is v m s^{-1}, where $v = 2t^2 - 4t + 5$.

(a) Find its acceleration when t = 4.

(b) Find the time when its acceleration is zero.

$a = \frac{dv}{dt} = 4t - 4$

(a) $t = 4$: $a = 4 \times 4 - 4$

(b) $4t - 4 = 0$, so $t = \frac{4}{4}$

Answer: (a) 12 m s^{-2} (b) 1 s

5 Natural logs and e^x (All boards)

Solve $e^{3x + 2} = 50$.

Give your answer to 3 significant figures.

Take natural logs: $3x + 2 = \ln 50 = 3.912$

$x = 0.637$

Answer: x = 0.637

6 Exponential graph through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve $y = ab^x$, where $b > 0$.

Find the values of a and b.

Through (0, 2): $a \times b^0 = 2$, so $a = 2$.

Through (2, 8): $2b^2 = 8$

$b^2 = 4$, so $b = 2$ ($b > 0$)

Answer: a = 2, b = 2

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7 Rough slope: limiting equilibrium or sliding (All boards)

A brick is on the point of sliding down a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{12}{5}$. Find the coefficient of friction.

Perpendicular: $R = mg\cos\alpha$
Parallel (limiting): $F = mg\sin\alpha$
$\mu = \frac{F}{R} = \tan\alpha$

Answer: $\frac{12}{5}$

8 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that it rains is 0.5. If so, the probability that Sam is late is 0.5; otherwise it is 0.1. Given that Sam is late, find the probability that it rains.

P(both) = 0.5 × 0.5 = 0.25; P(not first and second) = 0.5 × 0.1 = 0.05
P(second) = 0.25 + 0.05 = 0.3
P = 0.25 ÷ 0.3

Answer: $\frac{5}{6} = 0.833$

9 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $\sqrt{50}$ in the form $p\sqrt{q}$, where q is an integer as small as possible.

$\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2}$
$= 5\sqrt{2}$

Answer: $5\sqrt{2}$

10 Product rule (All boards)

Use the product rule to differentiate
$y = e^{2x}\sin 2x$.

$u = e^{2x}$, $v = \sin 2x$
$u' = 2e^{2x}$, $v' = 2\cos 2x$
$dy/dx = u'v + uv'$, which simplifies to the answer below

Answer: $dy/dx = e^{2x}(2\sin 2x + 2\cos 2x)$