Day 178 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Exact trigonometric values (All boards)

Write down the exact value of tan 150°.

tan 150° has the same size as tan 30°, and the sign comes from the quadrant (CAST).

tan 30° = $\frac{\sqrt{3}}{3}$, so tan 150° = $-\frac{\sqrt{3}}{3}$

Answer: $\mathbf{-\frac{\sqrt{3}}{3}}$

2 The remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the remainder when $-2x^3 + 3x^2 + 2x + 6$ is divided by $(3x - 5)$.

Put $3x - 5 = 0$: $x = \frac{5}{3}$

Remainder = $f(\frac{5}{3}) = (-\frac{250}{27}) + \frac{25}{3} + \frac{10}{3} + 6$

$= \frac{227}{27}$

Answer: $\mathbf{\frac{227}{27}}$

3 Definite integrals (exact) (All boards)

Evaluate

$\int_0^3 (x^3 + 3)\,dx$.

$[\frac{x^4}{4} + 3x]$ from 0 to 3

$= \frac{81}{4} + 9 = \frac{117}{4}$

Answer: $\mathbf{\frac{117}{4}}$

4 Free fall: dropped from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A brick is dropped from rest from the top of a tower of height 122.5 m. Take g = 9.8 m s^{-2} and ignore air resistance. Find (a) the time taken to reach the ground, (b) its speed on hitting the ground.

(a) $s = ut + \frac{1}{2}at^2$: $122.5 = \frac{1}{2} \times 9.8 \times t^2$

$122.5 = 4.9t^2$, so $t^2 = 25$

(b) $v = u + at = 9.8 \times 5$

Answer: (a) 5 s (b) 49 m $\mathbf{s^{-1}}$

5 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

20 pairs of data on daily mean temperature and sunshine hours have r = 0.44.
Test, at the 5% significance level, for evidence of positive correlation. The critical value is 0.3783.

H_0: $\rho = 0$, H_1: $\rho > 0$

r = 0.44 is beyond the critical value 0.3783

Reject H_0

Answer: Reject $\mathbf{H_0}$: there is evidence of positive correlation

6 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|2x + 3| = 5$.

$2x + 3 = 5$ or $2x + 3 = -5$

$x = 1$ or $x = -4$

Answer: $\mathbf{x = 1}$ or $\mathbf{x = -4}$

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7 Maximum friction on a horizontal surface (All boards)

A box of mass 4 kg rests on rough horizontal ground. The coefficient of friction is 0.2.
Taking g = 9.8 m s^{-2}, find the maximum friction force that can act on the box.

R = mg = 4 × 9.8 = 39.2
F_{max} = μR = 0.2 × 39.2
Answer: 7.84 N

8 Product rule (All boards)

Use the product rule to differentiate
$y = x^1 \sin 3x$.

u = x^1, v = sin 3x
u′ = 1, v′ = 3 cos 3x
dy/dx = u′v + uv′, which simplifies to the answer below
Answer: dy/dx = sin 3x + 3x cos 3x

9 Binomial test using a given probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(12, p). H_0: p = 0.4, H_1: p < 0.4.
In a sample, x = 2. It is given that P(X ≤ 2) = 0.0834.
Test H_0 at the 10% significance level.

Compare 0.0834 with 0.1
0.0834 < 0.1, so reject H_0
Answer: Reject H_0: evidence that p is less than 0.4

10 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $\sqrt{108}$ in the form $p\sqrt{q}$, where q is an integer as small as possible.

$\sqrt{108} = \sqrt{36 \times 3} = \sqrt{36} \times \sqrt{3}$
$= 6\sqrt{3}$
Answer: $6\sqrt{3}$