Day 176 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Interpreting the PMCC (AQA, Edexcel, OCR, MEI, Eduqas)

For data set A, r = 0.48. For data set B, r = 0.88.
Which data set has the stronger linear correlation? Give a reason.

Strength depends on the size of r, ignoring the sign.
|0.48| = 0.48 and |0.88| = 0.88.
Answer: data set B, as |r| is larger

2 Arithmetic sequences: nth term (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic sequence begins −5, −6, −7, −8, ...
Find (a) an expression for the nth term,
(b) the 39th term.

$a = -5$, $d = -1$
$u_n = a + (n - 1)d = -5 + (n - 1)(-1) = -n - 4$
$u_{39} = -43$
Answer: (a) $-n - 4$ (b) -43

3 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A systematic sample of 24 students is taken from a numbered list of 144.
The first one selected is number 5. Find the number of the 7th one selected.

Interval = 144 ÷ 24 = 6
Number = 5 + 6 × 6
Answer: 41

4 Two objects moving towards each other (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two cyclists start at the same time from points P and Q, 60 m apart, and cycle directly towards each other at constant speeds of 4 m s^{-1} and 8 m s^{-1}. Find the time until they meet.

Closing speed = 4 + 8 = 12 m s^{-1}
Time = $\frac{60}{12}$
Answer: 5 s

5 Solve with a shifted angle (All boards)

Solve $\cos(x + 20°) = \frac{\sqrt{3}}{2}$ for $0° \leq x \leq 360°$.

Let $u = x + 20°$, so $20° \leq u \leq 380°$
$\cos u = \frac{\sqrt{3}}{2}$: $u = 30°, 330°$
$x = u - 20°$: $x = 10°, 310°$
Answer: $x = 10°, 310°$

6 Arithmetic series: sum of terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 23 terms of the series
4 + 12 + 20 + 28 + ...

$a = 4$, $d = 8$, $n = 23$
$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{23}{2}[2(4) + 22 \times 8]$
$= 2116$
Answer: 2116

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7 Chain rule: gradient at a point (All boards)

Find the gradient of the curve $y = \sqrt{2x + 5}$ at the point where $x = 2$.

Use the chain rule: $dy/dx = \frac{2}{2\sqrt{2x + 5}}$

At $x = 2$: $dy/dx = \frac{1}{3}$

Answer: $\mathbf{\frac{1}{3}}$

8 Partial fractions (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

Express $\frac{7x - 6}{(x - 2)(x + 2)}$ in partial fractions.

Write $\frac{7x - 6}{(x - 2)(x + 2)} = \frac{A}{(x - 2)} + \frac{B}{(x + 2)}$

$7x - 6 = A(x + 2) + B(x - 2)$

$x = 2$: $8 = A(4)$ so $A = 2$

$x = -2$: $-20 = B(-4)$ so $B = 5$

Answer: $\mathbf{\frac{2}{(x - 2)} + \frac{5}{(x + 2)}}$

9 Integrate using partial fractions (All boards)

$\frac{5x + 12}{(x + 2)(x + 3)} \equiv \frac{A}{x + 2} + \frac{B}{x + 3}$.

Find A and B, and hence find $\int \frac{5x + 12}{(x + 2)(x + 3)}\, dx$.

$5x + 12 \equiv A(x + 3) + B(x + 2)$

$x = -2$ gives $A = 2$; $x = -3$ gives $B = 3$

$\int \left(\frac{2}{x + 2} + \frac{3}{x + 3}\right) dx = 2 \ln |x + 2| + 3 \ln |x + 3| + c$

Answer: A = 2, B = 3; 2 ln |x + 2| + 3 ln |x + 3| + c

10 Average speed over two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A runner travels 18 m at a constant speed of 6 m s^{-1}, then a further 80 m at a constant speed of 16 m s^{-1}. Find the average speed for the whole journey.

Time for first stage = $18 \div 6 = 3$ s

Time for second stage = $80 \div 16 = 5$ s

Average speed = $\frac{\text{total distance}}{\text{total time}} = \frac{98}{8}$

Answer: 12.25 m s^{-1}