Day 174 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Iteration $x_{n+1} = g(x_n)$ (All boards)

$x_{n+1} = (3x_n + 5)^{\frac{1}{3}}$, with $x_0 = 2$.
Find x_1, x_2 and x_3 to 3 decimal places.

$x_1 = 2.2240$
$x_2 = 2.2684$
$x_3 = 2.2770$
Answer: $x_1 = 2.224$, $x_2 = 2.268$, $x_3 = 2.277$

2 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

f(x) = 5x − 4 and g(x) = 2x + 2.
Find the value of gf(0).

f(0) = −4
g(−4) = −6
Answer: −6

3 Standard deviation from raw data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Here are 6 values: 12, 17, 13, 16, 14, 17
Find the mean and the standard deviation, to 3 s.f.

$\Sigma x = 89$, $\Sigma x^2 = 1343$, $n = 6$
Mean = 89 ÷ 6 = 14.83
Variance = $\frac{1343}{6} - (14.83)^2 = 3.806$
Standard deviation = $\sqrt{3.806}$
Answer: mean = 14.8, standard deviation = 1.95

4 Vector equation of a line (3D) (CIE)

A line has equation $\mathbf{r} = \begin{pmatrix}0\\2\\3\end{pmatrix} + t\begin{pmatrix}3\\-2\\1\end{pmatrix}$.
Find the coordinates of the point on the line where t = 3.

$$\mathbf{r} = \begin{pmatrix}0\\2\\3\end{pmatrix} + 3\begin{pmatrix}3\\-2\\1\end{pmatrix}$$
$$= \begin{pmatrix}9\\-4\\6\end{pmatrix}$$
Answer: (9, −4, 6)

5 Energy: falling object (potential and kinetic energy) (CIE)

A particle of mass 5 kg is released from rest and falls 78.4 m. Take g = 9.8 m s^{-2}. Use energy to find (a) the loss in potential energy (3 s.f.), (b) the speed of the particle after falling 78.4 m.

(a) Loss in PE = mgh = 5 × 9.8 × 78.4 = 3841.6
(b) $\frac{1}{2} \times 5 \times v^2 = 3841.6$
$v^2 = 1536.64$
Answer: (a) 3840 J (b) 39.2 m s^{-1}

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6 Straight line from logarithmic graph (All boards)

Variables x and y satisfy $y = ax^n$. A graph of $\log_{10} y$ against $\log_{10} x$ is a straight line with gradient $\frac{3}{2}$ and intercept 1.1. Find n and a, giving a to 3 significant figures.

$\log y = \log a + n \log x$

Gradient $n = \frac{3}{2}$

$\log a = 1.1$ so $a = 10^{1.1} = 12.6$

Answer: $n = \frac{3}{2}$, $a = 12.6$

7 Outliers (Edexcel, OCR, MEI)

For a data set $Q_1 = 36$ and $Q_3 = 52$.
A value is an outlier if it is more than 1.5 × IQR above Q_3 or below Q_1.
Determine whether 81 and 82 are outliers.

IQR = 52 − 36 = 16, so 1.5 × IQR = 24
Lower limit = 36 − 24 = 12; upper limit = 52 + 24 = 76
81 is an outlier; 82 is an outlier
Answer: 81 is an outlier; 82 is an outlier

8 Equation of a reciprocal graph from asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the graph of $y = \frac{k}{x - a} + c$ with its asymptotes. Find the values of a, c and k.

The asymptotes are x = 3 and y = −1, so a = 3 and c = −1.

Through (0, −2): $-2 = \frac{k}{0 - 3} + -1$

$\frac{k}{-3} = -1$, so k = 3

Answer: a = 3, c = −1, k = 3

9 Two particles over a smooth pulley (All boards)

Two particles of masses 3 kg and 4 kg hang from the ends of a light inextensible string over a smooth pulley and are released from rest. Take g = 9.8 m s^{-2}. Find the acceleration and the tension in the string.

4 kg: 4 × 9.8 − T = 4 a
3 kg: T − 3 × 9.8 = 3 a
Adding: 9.8 = 7 a, then T = 3(9.8 + a)
Answer: a = 1.4 m s^{-2}, T = 33.6 N

10 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $\frac{8 + \sqrt{2}}{\sqrt{2}}$ in the form $p\sqrt{2} + q$.

$\frac{8 + \sqrt{2}}{\sqrt{2}} = \frac{8}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{2}}$

$= 4\sqrt{2} + 1$

Answer: $4\sqrt{2} + 1$