Day 173 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:

(a) $2e^x + 5\ln x$

(b) $2\ln x$

(a) $dy/dx = 2e^x + \frac{5}{x}$

(b) $dy/dx = \frac{2}{x}$

Answer: (a) $2e^x + \frac{5}{x}$ (b) $\frac{2}{x}$

2 Midpoint and length of a line segment (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (6, 3) and B is (−2, −7).

Find (a) the midpoint of AB,

(b) the length of AB, as a surd in simplest form.

(a) Midpoint $= (\frac{6+(-2)}{2}, \frac{3+(-7)}{2}) = (2, -2)$

(b) $AB^2 = (-8)^2 + (-10)^2 = 64 + 100 = 164$

$AB = \sqrt{164} = 2\sqrt{41}$

Answer: (a) (2, −2) (b) $2\sqrt{41}$

3 Inverse trig functions (Edexcel, OCR)

Find the exact value, in radians, of

(a) arccos(−1) (b) arccos(0)

(a) $y = \arccos(-1)$ means $\cos y = -1$ with $0 \leq y \leq \pi$: $y = \pi$

(b) $y = \arccos(0)$ means $\cos y = 0$ with $0 \leq y \leq \pi$: $y = \frac{\pi}{2}$

Answer: (a) π (b) $\frac{\pi}{2}$

4 Straight line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find an equation of the line through A(−3, −2) and B(−6, −4), in the form ax + by + c = 0, where a, b and c are integers.

Gradient $= \frac{-4-(-2)}{-6-(-3)} = \frac{2}{3}$

$y - (-2) = \frac{2}{3}(x - (-3))$

Multiply out and rearrange to integer coefficients.

Answer: $2x - 3y = 0$

5 Lift: tension in the cable (All boards)

A lift of mass 200 kg is raised by a vertical cable with constant acceleration 1 m s^{-2} upwards. Take g = 9.8 m s^{-2}.

Find the tension in the cable.

Resolving upwards: $T - 200 \times 9.8 = 200 \times 1$

$T = 1960 + 200$

Answer: 2160 N

6 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(40, p)$. H_0: $p = 0.2$, H_1: $p > 0.2$.

Find the critical region for a test at the 5% significance level.

$P(X \geq 13) = 0.0432 \leq 0.05$

$P(X \geq 12) = 0.0875 > 0.05$

Answer: $X \geq 13$

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7 Integration by substitution (All boards)

Use $u = x^2 + 3$ to evaluate

$\int_0^1 2x(x^2 + 3)^2 \, dx$.

$u = x^2 + 3$, $du = 2x\,dx$; $x = 0$ gives $u = 3$, $x = 1$ gives $u = 4$

$\int_3^4 u^2 \, du = [\frac{u^3}{3}]$ from 3 to 4

$= \frac{4^3 - 3^3}{3}$

Answer: $\frac{37}{3}$

8 Position of a point relative to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $(x - 4)^2 + y^2 = 25$.
Determine whether the point $P(-1, -2)$ is inside, outside or on the circle.

Distance^2 from the centre (4, 0) to P:
$(-1 - 4)^2 + (-2 - 0)^2 = 25 + 4 = 29$
$r^2 = 25$, and $29 > 25$.
Answer: P is outside the circle

9 Moments: reactions on a uniform beam (AQA, Edexcel, OCR, MEI)

A uniform beam AB of mass 20 kg and length 5 m rests horizontally on supports at A and B. A load of mass 10 kg is placed on the beam 1.25 m from A. Take $g = 9.8$ m s^{-2}. Find the reaction at B.

Moments about A:
$R_B \times 5 = 20 \times 9.8 \times 2.5 + 10 \times 9.8 \times 1.25$
$R_B \times 5 = 612.5$
Answer: 122.5 N

10 Normal approximation to the binomial (CIE)

$X \sim B(150, 0.5)$. Use a normal approximation to find $P(X \geq 78)$, to 3 s.f.

$np = 75$ and $npq = 37.5$, so use $Y \sim N(75, 37.5)$
Continuity correction: $P(Y > 77.5)$
$z = (77.5 - 75) \div \sqrt{37.5}$
Answer: 0.342