Day 172 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Friction: acceleration along rough ground (All boards)

A block of mass 4 kg is pulled along rough horizontal ground by a horizontal force of 19.8 N.
The coefficient of friction is 0.25. Take g = 9.8 m s^{-2}. Find the acceleration.

R = 4 × 9.8 = 39.2
F = μR = 0.25 × 39.2 = 9.8
P − F = ma: 19.8 − 9.8 = 4 a
Answer: 2.5 m s^{-2}

2 Trapezium rule: strips shown (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows the curve $y = \sqrt{x + 4}$ from x = 0 to x = 3 divided into 3 strips of equal width.
Use the trapezium rule with these strips to estimate the shaded area to 3 s.f.

h = 1; y values: 2.000, 2.236, 2.449, 2.646
Area $\approx \frac{1}{2}[2.000 + 2.646 + 2(4.686)]$
= 7.01
Answer: 7.01

3 Straight line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find an equation of the line through A(−4, −6) and B(−5, 1), in the form ax + by + c = 0, where a, b and c are integers.

Gradient $= \frac{1 - (-6)}{-5 - (-4)} = -7$
y − (−6) = −7(x − (−4))
Multiply out and rearrange to integer coefficients.
Answer: 7x + y + 34 = 0

4 Integrating acceleration to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its acceleration at time t seconds is (4t + 5) m s^{-2}.
When t = 0 its velocity is 1 m s^{-1}.
Find its velocity when t = 3.

$v = \int a\,dt = 2t^2 + 5t + c$
t = 0, v = 1 gives c = 1
When t = 3: v = 18 + 15 + 1
Answer: 34 m s^{-1}

5 Integration by substitution (All boards)

Use the substitution $u = x^2 + 1$ to find
$\int 2x\,(x^2 + 1)^2\,dx$.

$u = x^2 + 1$, so $du = 2x\,dx$
$\int 1\,u^2\,du = \frac{1\,u^3}{3}$
Replace u: the answer below
Answer: $\frac{1(x^2 + 1)^3}{3} + c$

6 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(30, p). H_0: p = 0.4, H_1: p < 0.4.
Find the critical region for a test at the 5% significance level.

P(X ≤ 7) = 0.0435 ≤ 0.05
P(X ≤ 8) = 0.0940 > 0.05
Answer: X ≤ 7

Day 172 **A Level Maths: C Tier 10-a-day: Model Solutions**

7 Quadratic in sin or cos (All boards)

Solve $2\cos^2 x - 3\cos x + 1 = 0$ for $0° \leq x \leq 360°$.

Factorise: $(2\cos x - 1)(\cos x - 1) = 0$ (as a quadratic in cos x)

$\cos x = \frac{1}{2}$ or $\cos x = 1$

$x = 0°, 60°, 300°, 360°$

Answer: $x = 0°, 60°, 300°, 360°$

8 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-x^2 + 6x - 8 > 0$.

Solve $-x^2 + 6x - 8 = 0$: $-(x - 2)(x - 4) = 0$

Critical values $x = 2$ and $x = 4$.

The graph is an ∩-shaped parabola, so take the region between the roots.

Answer: $2 < x < 4$

9 Normal approximation to the binomial (CIE)

$X \sim B(60, 0.25)$. Use a normal approximation to find $P(X \geq 13)$, to 3 s.f.

$np = 15$ and $npq = 11.25$, so use $Y \sim N(15, 11.25)$

Continuity correction: $P(Y > 12.5)$

$z = (12.5 - 15) \div \sqrt{11.25}$

Answer: 0.772

10 The factor theorem and factorising (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 + x^2 - 22x - 40$

(a) Show that $(x + 4)$ is a factor of f(x).

(b) Factorise f(x) completely.

$f(-4) = (-64) + 16 + 88 + (-40) = 0$, so $(x + 4)$ is a factor.

$f(x) = (x + 4)(x^2 - 3x - 10)$

$x^2 - 3x - 10 = (x + 2)(x - 5)$

Answer: (b) $f(x) = (x + 4)(x + 2)(x - 5)$