Day 171 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Evaluating fractional and negative indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $100^{-\frac{1}{2}}$.

$100^{-\frac{1}{2}} = \frac{1}{100^{\frac{1}{2}}}$

$100^{\frac{1}{2}} = 10$, so $100^{\frac{1}{2}} = 10^1 = 10$

$= \frac{1}{10}$

Answer: $\mathbf{\frac{1}{10}}$

2 Hypotheses for a binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A shop says that a customer pays by card with probability 0.3.
A researcher thinks that the probability is different. Let p be the probability that a customer pays by card.
Write down suitable hypotheses for a test on p.

H_0 states the claimed value; H_1 matches the researcher's belief.

Two-tailed test

Answer: $\mathbf{H_0: p = 0.3, H_1: p \neq 0.3}$

3 Tension in a string supporting a hanging mass (All boards)

A particle of mass 2 kg hangs from a light string and is raised vertically with acceleration 1.5 m s^{-2}.
Taking g = 9.8 m s^{-2}, find the tension.

$T - mg = ma$

$T = 2 \times 9.8 + 2 \times 1.5$

Answer: 22.6 N

4 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=1}^{13} (6r + 8)$.

This is an arithmetic series with first term 14 and common difference 6.

$S_{13} = \frac{13}{2}[2 \times 14 + 12 \times 6]$

$= 650$

Answer: 650

5 Partial fractions: find the constants (All boards)

Express $\frac{-x}{(x + 2)(x + 3)}$ in the form $\frac{A}{x + 2} + \frac{B}{x + 3}$.
Find the values of A and B.

$-x \equiv A(x + 3) + B(x + 2)$

$x = -2$: $2 = A(1)$, so $A = 2$

$x = -3$: $3 = B(-1)$, so $B = -3$

Answer: $\mathbf{A = 2, B = -3}$

6 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is $(-6, -1)$ and B is $(-2, -7)$.
Find the equation of the perpendicular bisector of AB, in the form ax + by = c with integer a, b and c.

Midpoint of AB = $(-4, -4)$

Gradient of AB = $\frac{-6}{4} = -\frac{3}{2}$, so perpendicular gradient = $\frac{2}{3}$

$y - (-4) = \frac{2}{3}(x - (-4))$

Multiply out and rearrange.

Answer: $\mathbf{2x - 3y = 4}$

Day 171 **A Level Maths: C Tier 10-a-day: Model Solutions**

7 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 4^2)$. H_0: $\mu = 130$, H_1: $\mu > 130$.

A sample of 9 has mean $\bar{x} = 131.6$.
Test at the 5% level. Critical value: $z = 1.645$.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (131.6 - 130) \div (4 \div 3)$
$z = 1.20$
1.20 is outside the critical region, so do not reject H_0
Answer: $z = 1.20$; do not reject H_0

8 Pythagorean identity: find cos and tan (All boards)

θ is acute and $\cos\theta = \frac{40}{41}$.

Find the exact values of $\sin\theta$ and $\tan\theta$.

$\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{1600}{1681} = \frac{81}{1681}$

θ is acute, so $\sin\theta = \frac{9}{41}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{9}{40}$

Answer: $\sin\theta = \frac{9}{41}$, $\tan\theta = \frac{9}{40}$

9 Differentiating displacement to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A body moves in a straight line. Its displacement from O at time t seconds is s metres, where $s = 2t^3 + 5t^2 - 4t$.
Find its velocity when $t = 1$.

$v = \frac{ds}{dt} = 6t^2 + 10t - 4$

When $t = 1$: $v = 6 \times 1^2 + 10 \times 1 - 4$

Answer: 12 m s^{-1}

10 Amplitude and period from a graph (diagram) (All boards)

The diagram shows the graph of $y = a \sin bx$ for $0° \leq x \leq 360°$, where a and b are positive integers.
Find a and b.

The maximum value of y is 3, so $a = 3$

The graph completes 2 full cycles in 360°, so the period is 180° and $b = \frac{360}{180} = 2$

Answer: $a = 3$, $b = 2$