Day 170 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Laws of logarithms: single log (All boards)

Write 3 log 2 + log 7 − log 2 as a single logarithm, giving it in the form log N where N is an integer.

$3 \log 2 = \log 2^3 = \log 8$

$\log 8 + \log 7 - \log 2 = \log \frac{8 \times 7}{2} = \log 28$

Answer: log 28

2 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{1}{x - 3} - 4$, $x \neq 3$.

Find the range of f.

$\frac{1}{x - 3}$ takes every real value except 0.

Adding −4: f(x) takes every value except −4.

Answer: f(x) takes all real values except −4

3 Starting from rest: find the time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A skier starts from rest and moves in a straight line with constant acceleration 2 m s^{-2}. It travels 49 m. Find the time taken.

$s = ut + \frac{1}{2}at^2$ with $u = 0$

$49 = \frac{1}{2} \times 2 \times t^2$

$t^2 = 49$

Answer: 7 s

4 Conditional probability formula (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

P(B) = 0.9 and P(A | B) = 0.5.

Find P(A ∩ B).

$P(A \cap B) = P(A \mid B) \times P(B)$

$= 0.5 \times 0.9$

Answer: 0.45

5 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Divide $f(x) = x^4 - 3x^2 + 5x - 10$ by $(x - 1)$.

Find the quotient and the remainder.

Use synthetic division with a = 1:

Bottom row: 1, 1, −2, 3, −7

Quotient: $x^3 + x^2 - 2x + 3$

Remainder: −7

Answer: Quotient $x^3 + x^2 - 2x + 3$, remainder −7

6 Area between a line and a curve (All boards)

The curve $y = x^2$ and the line $y = 5x$ meet at the origin and at P.

Find the area enclosed between them.

$x^2 = 5x$ gives $x = 0$ or $x = 5$

Area $= \int_0^5 (5x - x^2)\,dx = \left[\frac{5x^2}{2} - \frac{x^3}{3}\right]$ from 0 to 5

$= \frac{5^3}{2} - \frac{5^3}{3} = \frac{125}{6}$

Answer: $\frac{125}{6}$

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7 Equation of a reciprocal graph from asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the graph of $y = \frac{k}{x - a} + c$ with its asymptotes. Find the values of a, c and k.

The asymptotes are $x = -1$ and $y = 2$, so $a = -1$ and $c = 2$.

Through $(0, -1)$: $-1 = \frac{k}{0 - (-1)} + 2$

$\frac{k}{1} = -3$, so $k = -3$

Answer: $a = -1$, $c = 2$, $k = -3$

8 Combined mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mean of 20 ages of the men is 40. The mean of 30 ages of the women is 37.
Find the mean of all 50 values.

Total of first group = $20 \times 40 = 800$
Total of second group = $30 \times 37 = 1110$
Mean = $1910 \div 50$
Answer: 38.2

9 Horizontal projection from a cliff (AQA, Edexcel, OCR, MEI)

A ball is thrown horizontally at 8 m s^{-1} from the top of a vertical cliff of height 44.1 m above the sea. Take g = 9.8 m s^{-2}.
Find (a) the time taken to reach the sea, (b) the horizontal distance of the landing point from the cliff.

(a) Vertically: $u = 0$, $s = 44.1$, $a = 9.8$

$44.1 = \frac{1}{2} \times 9.8 \times t^2$, so $t^2 = 9$

(b) Horizontal distance = speed × time = 8×3
Answer: (a) 3 s (b) 24 m

10 Stationary points of a cubic (All boards)

$y = x^3 - 27x$.
Find the coordinates of the stationary points and determine their nature.

$dy/dx = 3x^2 - 27 = 3(x + 3)(x - 3) = 0$
$x = -3$: $y = 54$; $x = 3$: $y = -54$
$d^2y/dx^2 = 6x + 0$: at $x = -3$ it is -18 (maximum), at $x = 3$ it is 18 (minimum)
Answer: $(-3, 54)$ maximum; $(3, -54)$ minimum