Day 17 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Addition rule (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For events A and B, P(A) = 0.45, P(B) = 0.5 and P(A ∪ B) = 0.65.
(a) Find P(A ∩ B).
(b) Find P(A′ ∩ B′).

(a) P(A ∩ B) = P(A) + P(B) − P(A ∪ B) = 0.45 + 0.5 − 0.65 = 0.3
(b) A′ ∩ B′ is outside both, so P = 1 − P(A ∪ B) = 1 − 0.65
Answer: (a) 0.3 (b) 0.35

2 Gradient and parallel or perpendicular lines (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line y = 2x + 3 is perpendicular to the line
y = kx − 5. Find the value of k.

Perpendicular lines have gradients with product −1.
2 × k = −1
Answer: k = $-\frac{1}{2}$

3 Integrate (ax + b)^n and related (All boards)

Find $\int (4x + 3)^3 \, dx$.

Reverse the chain rule: raise the power by 1, divide by the new power and by 4
Here a = 4
Answer: $\frac{(4x + 3)^4}{16} + c$

4 Equilibrium of a particle under three forces (diagram) (All boards)

A particle is in equilibrium under the three forces shown. cos α = $\frac{12}{13}$. Find P and Q.

Horizontally: P = 52 cos α = 52 × $\frac{12}{13}$

Vertically: Q = 52 sin α = 52 × $\frac{5}{13}$

Answer: P = 48, Q = 20

5 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $3x^2 + 24x + 43$ in the form $a(x + p)^2 + q$.

Take out a factor 3: $3(x^2 + 8x + \frac{43}{3})$

$x + 8 = (x + 4)^2 - 16$
$3[(x + 4)^2 - 16] + 43 = 3(x + 4)^2 - 5$
Answer: $3(x + 4)^2 - 5$

6 Angle between vectors (scalar product) (CIE)

a = −3**i** − 3**k** and b = −2**i** − 2**j**.
Find the angle between a and b, to 1 decimal place.

a.b = (−3)(−2) + (0)(−2) + (−3)(0) = 6
|a| = $\sqrt{18}$, |b| = $\sqrt{8}$
cos θ = $\frac{6}{\sqrt{18} \times \sqrt{8}}$, so θ = 60.0°
Answer: 60.0°

7 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{1}{x+5} + 4$, $x \neq -5$.

Find the range of f.

$\frac{1}{x+5}$ takes every real value except 0.

Adding 4: f(x) takes every value except 4.

Answer: f(x) takes all real values except 4

8 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(25, p)$. H_0: $p = 0.25$, H_1: $p > 0.25$.

Find the critical region for a test at the 1% significance level.

$P(X \geq 13) = 0.00337 \leq 0.01$

$P(X \geq 12) = 0.0107 > 0.01$

Answer: $X \geq 13$

9 Maximum speed from calculus (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds ($0 \leq t \leq 4$) is $v = (-t^2 + 4t)$ m s^{-1}. Find the maximum velocity and the time at which it occurs.

$a = \frac{dv}{dt} = 4 - 2t$

Maximum when $a = 0$: $t = \frac{4}{2} = 2$

$v = 4 \times 2 - 1 \times 2^2$

Answer: 4 m s^{-1} at t = 2

10 Optimisation (algebraic set-up) (All boards)

The product of x and $(12 - x^2)$ is $P = 12x - x^3$, $x > 0$.

Find the maximum value of P.

$dP/dx = 12 - 3x^2 = 0$, so $x^2 = 4$ and $x = 2$

$d^2P/dx^2 = -6x < 0$, so it is a maximum

$P = 12 \times 2 - 2^3 = 16$

Answer: P = 16