Day 169 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Evaluating fractional and negative indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $125^{\frac{1}{3}}$.

$125^{\frac{1}{3}} = (125^{\frac{1}{3}})^1$

$125^{\frac{1}{3}} = 5$

$(5)^1 = 5$

Answer: 5

2 Differentiating displacement to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves in a straight line. Its displacement from O at time t seconds is s metres, where $s = 2t^3 + t^2 + 2t$.
Find its velocity when t = 2.

$v = \frac{ds}{dt} = 6t^2 + 2t + 2$

When t = 2: $v = 6 \times 2^2 + 2 \times 2 + 2$

Answer: 30 m s^{-1}

3 Venn diagrams (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The Venn diagram shows probabilities for events A and B.
Find P(A | B).

$P(A \mid B) = P(A \cap B) \div P(B)$
$= 0.1 \div (0.1 + 0.25) = 0.1 \div 0.35$

Answer: $\frac{2}{7} = 0.286$

4 Pythagorean identity: find cos and tan (All boards)

θ is acute and $\sin\theta = \frac{8}{17}$.

Find the exact values of $\cos\theta$ and $\tan\theta$.

$\sin^2\theta + \cos^2\theta = 1$, so $\cos^2\theta = 1 - \frac{64}{289} = \frac{225}{289}$

θ is acute, so $\cos\theta = \frac{15}{17}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{8}{15}$

Answer: $\cos\theta = \frac{15}{17}$, $\tan\theta = \frac{8}{15}$

5 Horizontal projection from a cliff (AQA, Edexcel, OCR, MEI)

A ball is thrown horizontally at 13 m s^{-1} from the top of a vertical cliff of height 4.9 m above the sea. Take g = 9.8 m s^{-2}.
Find (a) the time taken to reach the sea, (b) the horizontal distance of the landing point from the cliff.

(a) Vertically: u = 0, s = 4.9, a = 9.8

$4.9 = \frac{1}{2} \times 9.8 \times t^2$, so $t^2 = 1$

(b) Horizontal distance = speed × time = 13 × 1

Answer: (a) 1 s (b) 13 m

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6 Newton-Raphson: one or two steps (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = \ln x + x - 4$.

Use the Newton-Raphson method with $x_0 = 3$ to find x_1 and x_2, giving your answer to 3 decimal places.

$f'(x) = \frac{1}{x} + 1$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 3 - \frac{0.09861}{1.333} = 2.9260$

$x_2 = 2.9260 - \frac{-0.0003090}{1.342} = 2.9263$

Answer: $x_1 = 2.926$, $x_2 = 2.926$

7 Real roots and the discriminant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of k for which $2x^2 - 6x + k = 0$ has no real roots.

$b^2 - 4ac < 0$

$36 - 8k < 0$

Answer: $k > \frac{9}{2}$

8 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a machine is set up wrongly is 0.3. If so, the probability that an item is faulty is 0.9; otherwise it is 0.5. Given that an item is faulty, find the probability that a machine is set up wrongly.

P(both) = 0.3 × 0.9 = 0.27; P(not first and second) = 0.7 × 0.5 = 0.35

P(second) = 0.27 + 0.35 = 0.62

P = 0.27 ÷ 0.62

Answer: $\frac{27}{62} = 0.435$

9 Solve with a shifted angle (All boards)

Solve $\sin(x - 40°) = -\frac{\sqrt{2}}{2}$ for $0° \le x \le 360°$.

Let $u = x - 40°$, so $-40° \le u \le 320°$

$\sin u = -\frac{\sqrt{2}}{2}$: $u = 225°, 315°$

$x = u + 40°$: $x = 265°, 355°$

Answer: $x = 265°, 355°$

10 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=1}^{11} (-r - 4)$.

This is an arithmetic series with first term -5 and common difference -1.

$S_{11} = \frac{11}{2}[2 \times (-5) + 10 \times (-1)]$

$= -110$

Answer: -110