Day 168 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Modelling assumptions (All boards)

In an exam question, a car is modelled as a particle.
State what this assumption means for the motion or forces in the model.

Modelling assumption: car as a particle.
The assumption means: t.he car's dimensions are ignored and rotation is not considered
Answer: The car's dimensions are ignored and rotation is not considered

2 Simplify using identities (All boards)

Simplify 3 sin x cos x ÷ cos x.

Cancel cos x
Answer: 3 sin x

3 Normal approximation to the binomial (CIE)

X ~ B(60, 0.2). Use a normal approximation to find P(X ≥ 9), to 3 s.f.

np = 12 and npq = 9.6, so use Y ~ N(12, 9.6)
Continuity correction: P(Y > 8.5)
$z = (8.5 - 12) \div \sqrt{9.6}$
Answer: 0.871

4 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{k=1}^{4} 2 \times 3^{k-1}$.

Geometric series: first term 2, common ratio 3, 4 terms.
$S_4 = \frac{2(3^4 - 1)}{3 - 1}$
$= 80$
Answer: 80

5 Binomial: modelling conditions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X is the number of faulty bulbs in a random sample of 11 from a very large batch, 4% being faulty.
Can X be modelled by a binomial distribution? Give a reason.

A binomial model needs: a fixed number of trials, two outcomes, constant probability, independent trials.
Here a fixed number of trials, independent (very large batch), constant probability 0.04.
Answer: Yes, a fixed number of trials, independent (very large batch), constant probability 0.04

6 Finding a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), where f(x) is a cubic. Find f(x) in the form k(x − a)(x − b)(x − c).

The curve crosses the x-axis at x = −3, −1 and 3.
f(x) = k(x + 3)(x + 1)(x − 3)
At (0, −27): k × 3 × 1 × (−3) = −27
k = 3
Answer: f(x) = 3(x + 3)(x + 1)(x − 3)

7 Trapezium rule: strips shown (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows the curve $y = \sqrt{x + 2}$ from x = 0 to x = 4 divided into 3 strips of equal width.
Use the trapezium rule with these strips to estimate the shaded area to 3 s.f.

h = 1.3333333333333333; y values: 1.414, 1.826, 2.160, 2.449
$\text{Area} \approx \frac{1.3333333333333333}{2}[1.414 + 2.449 + 2(3.986)]$
$= 7.89$
Answer: 7.89

8 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = (x - 3)^2 - 3$, $x \geq 0$.
Find the range of f.

The minimum of $(x - 3)^2$ is 0 at $x = 3$, which is allowed since $3 \geq 0$.
So $f(x) \geq -3$.
Answer: $f(x) \geq -3$

9 Double angle equation (All boards)

Solve $\sin 3x = \frac{1}{2}$ for $0° \leq x \leq 120°$.

Let $u = 3x$, so $0° \leq u \leq 360°$

$\sin u = \frac{1}{2}$: $u = 30°, 150°, 390°, 510°, 750°, 870°$

Divide by 3: $x = 10°, 50°$
Answer: $x = 10°, 50°$

10 Average speed over two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A boat travels 36 m at a constant speed of 6 m s^{-1}, then a further 100 m at a constant speed of 10 m s^{-1}. Find the average speed for the whole journey.

Time for first stage = 36 ÷ 6 = 6 s
Time for second stage = 100 ÷ 10 = 10 s

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{136}{16}$$

Answer: 8.5 m s^{-1}