Day 167 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Amplitude and period from a graph (diagram) (All boards)

The diagram shows the graph of $y = a \sin bx$ for $0° \leq x \leq 360°$, where a and b are positive integers.
Find a and b.

The maximum value of y is 5, so a = 5

The graph completes 3 full cycles in 360°, so the period is 120° and $b = \frac{360}{120} = 3$

Answer: a = 5, b = 3

2 Parametric equations to Cartesian form (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

A line has parametric equations
$x = t + 1$, $y = 4t - 2$.
Find its equation in the form $y = mx + c$.

From $x = t + 1$: $t = \frac{x - 1}{1}$

$y = 4 \times \frac{x - 1}{1} - 2$

Answer: $y = 4x - 6$

3 Regression: prediction and reliability (AQA, Edexcel, OCR, MEI, Eduqas)

For 7 data pairs, with advertising spend (£s hundreds) from 12 to 22, the regression line is $S = 15 + 6a$.
Estimate sales (£s thousands) when a = 10 and comment on the reliability.

$S = 15 + 6 \times 10 = 75$
10 is outside the range 12 to 22, so this is extrapolation.
Answer: S = 75; estimate is unreliable, as 10 is outside the data range (extrapolation)

4 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers x and y such that $16x + 4y = 25$.

Assume that integers x and y exist with $16x + 4y = 25$.
Then $4(4x + y) = 25$, so 25 would be a multiple of 4.
But $25 = 4 \times 6 + 1$, so it is not a multiple of 4.
This is a contradiction, so no such integers exist.
Answer: Proved

5 Reciprocal trig functions: exact values (All boards)

Write down the exact values of
(a) cot 150° (b) cosec 45°

(a) $\cot 150° = \frac{1}{\tan 150°} = \frac{1}{-\frac{\sqrt{3}}{3}} = -\sqrt{3}$

(b) $\operatorname{cosec} 45° = \frac{1}{\sin 45°} = \frac{1}{\frac{\sqrt{2}}{2}} = \sqrt{2}$

Answer: (a) $-\sqrt{3}$ (b) $\sqrt{2}$

6 Free fall: dropped from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A coin is dropped from rest from the top of a bridge of height 44.1 m. Take g = 9.8 m s^{-2} and ignore air resistance. Find (a) the time taken to reach the ground, (b) its speed on hitting the ground.

(a) $s = ut + \frac{1}{2}at^2$: $44.1 = \frac{1}{2} \times 9.8 \times t^2$

$44.1 = 4.9t^2$, so $t^2 = 9$
(b) $v = u + at = 9.8 \times 3$
Answer: (a) 3 s (b) 29.4 m s^{-1}

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7 Two-tailed binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(15, p). H_0: p = 0.3, H_1: p ≠ 0.3. In a sample, x = 10.

Test at the 10% significance level.

Two-tailed test, so compare with 0.1 ÷ 2 = 0.05

P(X ≥ 10) = 0.00365

0.00365 < 0.05, so reject H_0

Answer: Reject H_0 (P(X ≥ 10) = 0.00365)

8 Linear and quadratic simultaneous equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations:

$y = x^2 + 6x + 2$, $y = 3x + 6$.

$x^2 + 6x + 2 = 3x + 6$

$x^2 + 3x - 4 = 0$

$(x + 4)(x - 1) = 0$

Use $y = 3x + 6$ to find each y.

Answer: x = −4, y = −6 and x = 1, y = 9

9 Moments: rod on a pivot with a particle (AQA, Edexcel, OCR, MEI)

A uniform rod AB of mass 6 kg and length 5 m is supported by a pivot 1.25 m from A. A particle of mass m kg is attached at A and the rod rests horizontally in equilibrium. Find m.

Moments about the pivot:

m g × 1.25 = 6 g × (2.5 − 1.25)

$m = \frac{6 \times 1.25}{1.25}$

Answer: 6 kg

10 Rewrite a sin x + b cos x in R form (All boards)

Express 5 sin x + 12 cos x in the form R sin(x + α), where R > 0 and 0° < α < 90°.

Give α to 1 decimal place.

R sin(x + α) = R sin x cos α + R cos x sin α

R cos α = 5 and R sin α = 12

$R = \sqrt{5^2 + 12^2} = 13$, $\tan \alpha = \frac{12}{5}$, α = 67.4°

Answer: 13 sin(x + 67.4°)