Day 166 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Finding a quadratic from its turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of a quadratic curve.

Find its equation in the form $y = a(x - p)^2 + q$.

The turning point is $(-3, -1)$, so $y = a(x + 3)^2 - 1$.

The curve passes through $(0, 8)$:

$8 = a(3)^2 - 1$, so $9 = 9a$

$a = 1$

Answer: $y = (x + 3)^2 - 1$

2 F = ma: find the force or the mass (All boards)

A resultant force of 16 N gives a body an acceleration of 2 m s^{-2}. Find the mass of the body.

$F = ma$

$16 = m \times 2$

Answer: 8 kg

3 Integrate standard functions (All boards)

Find

(a) $\int \frac{3}{x}\,dx$

(b) $\int (3e^x + \frac{3}{x})\,dx$

Use the standard integrals and divide by the coefficient of x where needed.

(a) $= 3 \ln|x| + c$

(b) $= 3e^x + 3 \ln|x| + c$

Answer: (a) $3 \ln|x| + c$ (b) $3e^x + 3 \ln|x| + c$

4 Parametric equations to Cartesian form (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

A curve has parametric equations

$x = t - 1$, $y = t^2 - 3$.

Find a Cartesian equation of the curve.

From $x = t - 1$: $t = x + 1$

Substitute into y: $y = (x + 1)^2 - 3$

Answer: $y = (x + 1)^2 - 3$

5 Optimisation (algebraic set-up) (All boards)

The product of x and $(75 - x^2)$ is $P = 75x - x^3$, $x > 0$.

Find the maximum value of P.

$dP/dx = 75 - 3x^2 = 0$, so $x^2 = 25$ and $x = 5$

$d^2P/dx^2 = -6x < 0$, so it is a maximum

$P = 75 \times 5 - 5^3 = 250$

Answer: P = 250

6 Grouped data: estimated mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows grouped data.

Class: $20 \le x < 40$; $40 \le x < 60$; $60 \le x < 80$; $80 \le x < 100$

Frequency: 12, 3, 15, 15

Estimate the mean, to 3 s.f.

Midpoints: 30, 50, 70, 90

$\Sigma fx = 12 \times 30 + 3 \times 50 + 15 \times 70 + 15 \times 90 = 2910$

Mean $\approx 2910 \div 45$

Answer: 64.7

7 Identity then solve (sin² to cos) (All boards)

Solve $-4\sin^2 x + 6\cos x + 6 = 0$
for $0° \leq x \leq 360°$.

Use $\sin^2 x = 1 - \cos^2 x$: $4\cos^2 x + 6\cos x + 2 = 0$

Factorise and solve: $\cos x = -1$ or $\cos x = -\frac{1}{2}$

$x = 120°, 180°, 240°$

Answer: $x = 120°, 180°, 240°$

8 Maximum speed from calculus (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds ($0 \leq t \leq 6$) is $v = (-2t^2 + 12t)$ m s^{-1}. Find the maximum velocity and the time at which it occurs.

$a = \frac{dv}{dt} = 12 - 4t$

Maximum when $a = 0$: $t = \frac{12}{4} = 3$

$v = 12 \times 3 - 2 \times 3^2$

Answer: 18 m s^{-1} at $t = 3$

9 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a machine is set up wrongly is 0.6. If so, the probability that an item is faulty is 0.6; otherwise it is 0.4. Given that an item is faulty, find the probability that a machine is set up wrongly.

P(both) = 0.6 × 0.6 = 0.36; P(not first and second) = 0.4 × 0.4 = 0.16
P(second) = 0.36 + 0.16 = 0.52
P = 0.36 ÷ 0.52

Answer: $\frac{9}{13} = 0.692$

10 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = x + k$ is a tangent to the curve
$y = x^2 + 3x + 6$.
Find the value of k.

$x^2 + 3x + 6 = x + k$
$x^2 + 2x + (6 - k) = 0$
A tangent touches once: discriminant = 0
$(2)^2 - 4(6 - k) = 0$
Answer: $k = 5$