Day 165 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Integrate (ax + b)^n and related (All boards)

Find $\int \sin(5x + 3)\,dx$.

$\int \sin(ax + b)\,dx = -\frac{1}{a}\cos(ax + b)$

Here $a = 5$

Answer: $-\frac{1}{5}\cos(5x + 3) + c$

2 Expanding brackets with surds (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Expand and simplify $(5 + \sqrt{2})(5 - \sqrt{2})$.

$= 5^2 - (\sqrt{2})^2$
$= 25 - 2$
Answer: 23

3 Work done and kinetic energy (CIE)

A particle of mass 8 kg changes its speed from 3 m s^{-1} to 6 m s^{-1}. Find the increase in its kinetic energy.

$KE = \frac{1}{2}mv^2$

Increase $= \frac{1}{2} \times 8 \times 6^2 - \frac{1}{2} \times 8 \times 3^2$

Answer: 108 J

4 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = (x + 3)^2 + 3$, $x \geq 0$.
Find the range of f.

For $x \geq 0$ the least value is at $x = 0$ (the vertex $x = -3$ is not allowed).
$f(0) = 9 + 3 = 12$, and f increases for $x > 0$.
Answer: $f(x) \geq 12$

5 Rearrange and iterate (All boards)

(a) Show that $x^2 - x - 1 = 0$ can be rearranged to $x = 1 + \frac{1}{x}$.

(b) Use x_{n+1} = this formula with $x_0 = 2$ to find x_1, x_2 to 3 d.p.

(a) Divide $x^2 - x = 1$ by x: $x - 1 = \frac{1}{x}$, so $x = 1 + \frac{1}{x}$

(b) $x_1 = 1.5000$, $x_2 = 1.6667$
Answer: (a) shown (b) $x_1 = 1.500$, $x_2 = 1.667$

6 Intercepts of a cubic graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = 2(x + 2)(x + 1)(x - 5)$. Write down the coordinates of the points where the curve meets the axes.

$y = 0$: $x = -2, -1, 5$
$x = 0$: $y = 2 \times 1 \times (-5) \times 2 = -20$
Answer: $(-2, 0)$, $(-1, 0)$, $(5, 0)$, $(0, -20)$

7 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 6^2)$. H_0: $\mu = 90$, H_1: $\mu \neq 90$.

A sample of 16 has mean $\bar{x} = 87.6$.
Test at the 5% level. Critical value: $z = \pm 1.96$.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (87.6 - 90) \div (6 \div 4)$
$z = -1.60$
-1.60 is outside the critical region, so do not reject H_0
Answer: $z = -1.60$; do not reject H_0

8 Energy: falling object (potential and kinetic energy) (CIE)

A particle of mass 2 kg is released from rest and falls 4.9 m. Take g = 9.8 m s^{-2}. Use energy to find (a) the loss in potential energy (3 s.f.), (b) the speed of the particle after falling 4.9 m.

(a) Loss in PE = mgh = 2 × 9.8 × 4.9 = 96.04

(b) $\frac{1}{2} \times 2 \times v^2 = 96.04$

$v^2 = 96.04$

Answer: (a) 96.0 J (b) 9.8 m s^{-1}

9 Quadratic in sin or cos (All boards)

Solve $2\sin^2 x + \sin x = 0$ for $0° \leq x \leq 360°$.

Factorise: (2 sin x + 1)sin x = 0 (as a quadratic in sin x)

$\sin x = -\frac{1}{2}$ or sin x = 0

x = 0°, 180°, 210°, 330°, 360°

Answer: x = 0°, 180°, 210°, 330°, 360°

10 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A systematic sample of 30 customers is taken from a numbered list of 450.
The first one selected is number 12. Find the number of the 6th one selected.

Interval = 450 ÷ 30 = 15

Number = 12 + 5 × 15

Answer: 87