Day 164 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sequence is defined by $u_{n+1} = -2u_n + 6$, $u_1 = 5$.

Find u_2, u_3 and u_4.

$u_2 = (-2) \times 5 + 6 = -4$

$u_3 = (-2) \times (-4) + 6 = 14$

$u_4 = (-2) \times 14 + 6 = -22$

Answer: $u_2 = -4$, $u_3 = 14$, $u_4 = -22$

2 Constant acceleration: $v^2 = u^2 + 2as$ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line with constant acceleration. Its speed increases from 1 m s^{-1} to 15 m s^{-1} as it travels 112 m. Find the acceleration.

$v^2 = u^2 + 2as$

$15^2 = 1^2 + 2 \times a \times 112$

$224 = 224\ a$

Answer: 1 m s^{-2}

3 Discrete random variable: finding k (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The discrete random variable X has $P(X = x) = k(3x + 3)$ for $x = 1, 2, 3, 4, 5$.

(a) Find the value of k.

(b) Find $P(X \geq 3)$.

(a) Probabilities sum to 1: $k(6 + 9 + 12 + 15 + 18) = 1$

$k = \frac{1}{60}$

(b) $P(X \geq 3) = \frac{12}{60} + \frac{15}{60} + \frac{18}{60}$

Answer: (a) $k = \frac{1}{60}$ (b) $\frac{3}{4}$

4 Trapezium rule from a function (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 4 strips to estimate

$\int_0^6 2^x \, dx$,

giving your answer to 3 significant figures.

$h = \frac{6 - 0}{4} = 1.5$; $x = 0, 1.5, 3, 4.5, 6$

$y = 1.000, 2.828, 8.000, 22.63, 64.00$

Area $\approx \frac{h}{2}$[first + last + 2(sum of the rest)] $= \frac{1.5}{2}[1.000 + 64.00 + 2 \times 33.46]$

$= 98.9$

Answer: 98.9

5 Constant velocity vector: when does it reach a point? (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves with constant velocity $(-2\mathbf{i} + 3\mathbf{j})$ m s^{-1}. It is at the point with position vector $(4\mathbf{i})$ m at $t = 0$. Find the time when its position vector is $(-10\mathbf{i} + 21\mathbf{j})$ m.

$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$

$(-10\mathbf{i} + 21\mathbf{j}) = (4\mathbf{i}) + t(-2\mathbf{i} + 3\mathbf{j})$

Equating i components: $-10 = 4 + -2t$

Answer: 7 s

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6 Arithmetic series: smallest n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic series has first term 6 and common difference 5. Find the smallest n for which the sum of the first n terms exceeds 200.

$S_n = \frac{n}{2}[12 + 5(n - 1)] = \frac{n(5n + 7)}{2}$

$S_8 = 188$ and $S_9 = 234$

The sum first exceeds 200 when n = 9.

Answer: n = 9

7 Integration by substitution (All boards)

Use the substitution $u = x^2 + 1$ to find

$\int \frac{4x}{x^2 + 1}\,dx$.

$u = x^2 + 1$, $du = 2x\,dx$, so $x\,dx = \frac{1}{2}\,du$

$\int \frac{4}{2u}\,du = \frac{4}{2}\ln|u|$

Answer: $2\ln(x^2 + 1) + c$

8 Solving quadratics: surd answers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^2 - 10x + 3 = 0$, giving your answers in the form $p \pm \sqrt{q}$.

Complete the square: $(x - 5)^2 - 25 + 3 = 0$

$(x - 5)^2 = 22$

$x - 5 = \pm\sqrt{22}$

Answer: $x = 5 \pm \sqrt{22}$

9 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$x = 4\cos t$, $y = 5\sin t$.

Find dy/dx in terms of t, and its value when $t = \frac{\pi}{4}$.

$dx/dt = -4\sin t$, $dy/dt = 5\cos t$

$dy/dx = \frac{5\cos t}{-4\sin t} = -\frac{5}{4}\cot t$

At $t = \frac{\pi}{4}$, $\cot t = 1$, so $dy/dx = -\frac{5}{4}$

Answer: $dy/dx = -\frac{5}{4}\cot t$; $-\frac{5}{4}$

10 Independent events (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

P(A) = 0.5, P(B) = 0.6 and $P(A \cap B) = 0.3$.

(a) Find $P(A \mid B)$.

(b) Are A and B independent? Give a reason.

(a) $P(A \mid B) = P(A \cap B) \div P(B) = 0.3 \div 0.6 = 0.5$

(b) $P(A) \times P(B) = 0.5 \times 0.6 = 0.3$

This equals $P(A \cap B) = 0.3$, so A and B are independent

Answer: (a) 0.5 (b) independent, as $P(A) \times P(B) = P(A \cap B)$