Day 163 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $\frac{25 + \sqrt{5}}{\sqrt{5}}$ in the form $p\sqrt{5} + q$.

$\frac{25 + \sqrt{5}}{\sqrt{5}} = \frac{25}{\sqrt{5}} + \frac{\sqrt{5}}{\sqrt{5}}$

$= 5\sqrt{5} + 1$

Answer: $5\sqrt{5} + 1$

2 F = ma: find the force or the mass (All boards)

A resultant force of 15 N gives a body an acceleration of 3 m s^{-2}. Find the mass of the body.

F = ma

15 = m × 3

Answer: 5 kg

3 Laws of logarithms: single log (All boards)

Write 3 log 2 + log 3 − log 2 as a single logarithm,
giving it in the form log N where N is an integer.

3 log 2 = log 2^3 = log 8

log 8 + log 3 − log 2 = log $\frac{8 \times 3}{2}$ = log 12

Answer: log 12

4 Poisson distribution (CIE)

Flaws occur in a roll of cloth at an average rate of 3.5 per hour, at random.
Find the probability of exactly 12 in 3 hours, to 3 s.f.

Mean for 3 hours = 3 × 3.5 = 10.5

P(X = 12) = $e^{-10.5} \times 10.5^{12} \div 479001600$!

Answer: 0.103

5 Modulus graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch y = |2x − 8|. Write down the coordinates of the vertex and of the point where the graph meets the y-axis.

The vertex is on the x-axis where 2x − 8 = 0: x = 4

x = 0: y = |−8| = 8

Answer: Vertex (4, 0) ; y-axis (0, 8)

6 Separable differential equations (All boards)

Given that dy/dx = 3y and y = 3 when x = 0,
find y in terms of x.

$\frac{1}{y}$ dy = 3 dx, so ln y = 3x + c

x = 0, y = 3: c = ln 3

y = $e^{3x + \ln 3}$ = 3 e^{3x}

Answer: y = 3 e^{3x}

7 Binomial test using a given probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(30, p). H_0: p = 0.1, H_1: p > 0.1.

In a sample, x = 7. It is given that P(X ≥ 7) = 0.0258.

Test H_0 at the 5% significance level.

Compare 0.0258 with 0.05

0.0258 < 0.05, so reject H_0

Answer: Reject H_0: evidence that p is greater than 0.1

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8 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 - 3x)^{-3}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots$ with $n = -3$, $y = -3x$

$= 1 + (-3)(-3x) + \frac{(-3)((-4))}{2}(-3x)^2$

$= 1 + 9x + 54x^2$

Valid when $|-3x| < 1$, i.e. $|x| < \frac{1}{3}$

Answer: $1 + 9x + 54x^2$; $|x| < \frac{1}{3}$

9 Moments: rod on a pivot with a particle (AQA, Edexcel, OCR, MEI)

A uniform rod AB of mass 6 kg and length 4 m is supported by a pivot 0.5 m from A. A particle of mass m kg is attached at A and the rod rests horizontally in equilibrium. Find m.

Moments about the pivot:

$m\,g \times 0.5 = 6\,g \times (2 - 0.5)$

$m = \frac{6 \times 1.5}{0.5}$

Answer: 18 kg

10 Rearrange and iterate (All boards)

(a) Show that $x^3 - x - 7 = 0$ can be rearranged to $x = (x + 7)^{\frac{1}{3}}$.
(b) Use x_{n+1} = this formula with $x_0 = 2$ to find x_1, x_2 to 3 d.p.

(a) $x^3 = x + 7$, so $x = \sqrt[3]{(x + 7)}$

(b) $x_1 = 2.0801$, $x_2 = 2.0862$

Answer: (a) shown (b) $x_1 = 2.080$, $x_2 = 2.086$