Day 161 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sequence is defined by $u_{n+2} = u_{n+1} + u_n$, with
$u_1 = 2$ and $u_2 = 3$. Find u_6.

$u_3 = 3 + 2 = 5$
$u_4 = 5 + 3 = 8$, $u_5 = 8 + 5 = 13$
$u_6 = 13 + 8 = 21$
Answer: $u_6 = 21$

2 Amplitude and period from a graph (diagram) (All boards)

The diagram shows the graph of $y = a \sin bx$ for $0° \leq x \leq 360°$, where a and b are positive integers.
Find a and b.

The maximum value of y is 5, so a = 5

The graph completes 4 full cycles in 360°, so the period is 90° and $b = \frac{360}{90} = 4$

Answer: a = 5, b = 4

3 Differentiating velocity to find acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds is v m s^{-1}, where $v = 2t^2 - 16t + 6$.
(a) Find its acceleration when t = 3.
(b) Find the time when its acceleration is zero.

$a = \frac{dv}{dt} = 4t - 16$
(a) $t = 3$: $a = 4 \times 3 - 16$
(b) $4t - 16 = 0$, so $t = \frac{16}{4}$
Answer: (a) -4 m s^{-2} (b) 4 s

4 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|3x + 4| = 11$.

$3x + 4 = 11$ or $3x + 4 = -11$
$x = \frac{7}{3}$ or $x = -5$
Answer: $x = \frac{7}{3}$ or $x = -5$

5 Parallel vectors (All boards)

$\mathbf{a} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 20 \\ 8 \end{pmatrix}$.
Show that a and b are parallel and find the ratio |b| : |a|.

$\mathbf{b} = 4 \times \mathbf{a}$, since $4 \times 5 = 20$ and $4 \times 2 = 8$
So a and b are parallel and |b| : |a| = 4 : 1
Answer: b = 4 a, so parallel; ratio 4 : 1

6 Finding an unknown using the factor theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

When $x^3 + kx^2 - x + 4$ is divided by (x + 1), the remainder is 5. Find the value of k.

$f(-1) = 5$
$-1 + k + 5 = 5$
$k = 1$
Answer: k = 1

7 Stone thrown down from a cliff (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically downwards at 8 m s^{-1} from the top of a cliff and hits the sea after 4 s. Taking g = 9.8 m s^{-2}, find the height of the cliff.

$s = ut + \frac{1}{2}at^2$

$s = 8 \times 4 + \frac{1}{2} \times 9.8 \times 4^2$

$s = 32 + 78.4$

Answer: 110.4 m

8 Normal approximation to the binomial (CIE)

X ~ B(120, 0.25). Use a normal approximation to find P(X ≥ 25), to 3 s.f.

np = 30 and npq = 22.5, so use Y ~ N(30, 22.5)

Continuity correction: P(Y > 24.5)

$z = (24.5 - 30) \div \sqrt{22.5}$

Answer: 0.877

9 Exponential growth and decay model (All boards)

The number of bacteria is $N = 1000\, e^{0.02t}$, where t is the time in hours.

Find (a) N when t = 5, (b) the time when N = 2000.

Give answers to 3 s.f.

(a) $N = 1000\, e^{0.02 \times 5} = 1000\, e^{0.1} = 1110$

(b) $e^{0.02t} = 2$, so $0.02t = \ln 2$

$t = \frac{\ln 2}{0.02} = 34.7$ hours

Answer: (a) 1110 (b) 34.7 hours

10 Large data set: handling readings (AQA, Edexcel, OCR, MEI, Eduqas)

Seven daily rainfall readings (mm) from the Large Data Set are: 3.2, 4.6, 4.2, 0.4, tr, tr, 0.8.

Here 'tr' means a trace, which is treated as 0.

Find the mean daily rainfall to 3 s.f.

Replace each 'tr' by 0

Total = 13.2

Mean = 13.2 ÷ 7

Answer: 1.89 mm