Day 160 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Integration by parts (All boards)

Use integration by parts to find
$\int x \sin 2x \, dx$.

$u = x$, $dv/dx = \sin 2x$; $du/dx = 1$, $v = -\frac{\cos 2x}{2}$

$\int u \, (dv/dx) \, dx = uv - \int v \, (du/dx) \, dx$
Evaluate the remaining integral and simplify

Answer: $-\frac{\mathbf{x \cos 2x}}{\mathbf{2}} + \frac{\mathbf{\sin 2x}}{\mathbf{4}} + \mathbf{c}$

2 Position vector under constant velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves with constant velocity $(-3\mathbf{i} + 4\mathbf{j})$ m s^{-1}.
At time t = 0 its position vector is $(2\mathbf{i} + 5\mathbf{j})$ m.
(a) Find its position vector when t = 5.
(b) Find the distance it travels in 5 s.

(a) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = (2\mathbf{i} + 5\mathbf{j}) + 5(-3\mathbf{i} + 4\mathbf{j})$

(b) speed $= \sqrt{3^2 + 4^2} = 5$ m s^{-1}
distance $= 5 \times 5$
Answer: (a) $(-13\mathbf{i} + 25\mathbf{j})$ m (b) 25 m

3 Expanding brackets with surds (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Expand and simplify $(6 - \sqrt{5})^2$.

$= 6^2 - 2 \times 6 \times \sqrt{5} + (\sqrt{5})^2$
$= 36 - 12\sqrt{5} + 5$
Answer: $41 - 12\sqrt{5}$

4 Binomial: cumulative probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(18, 0.4). Find $P(X \leq 9)$, giving your answer to 3 s.f.

Using the cumulative binomial function (or adding $P(X = 0)$ up to $P(X = 9)$)
Answer: 0.865

5 Product rule (All boards)

Use the product rule to differentiate
$y = e^{3x} \sin 4x$.

$u = e^{3x}$, $v = \sin 4x$
$u' = 3e^{3x}$, $v' = 4 \cos 4x$
$dy/dx = u'v + uv'$, which simplifies to the answer below
Answer: $dy/dx = e^{3x}(3 \sin 4x + 4 \cos 4x)$

6 Finding a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), where f(x) is a cubic. Find f(x) in the form $k(x - a)(x - b)(x - c)$.

The curve crosses the x-axis at $x = -4$, -3 and 2.
$f(x) = k(x + 4)(x + 3)(x - 2)$
At (0, 48): $k \times 4 \times 3 \times (-2) = 48$
$k = -2$
Answer: $f(x) = -2(x + 4)(x + 3)(x - 2)$

7 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(15, p). H_0: p = 0.5, H_1: p < 0.5.
Find the critical region for a test at the 1% significance level.

$P(X \leq 2) = 0.00369 \leq 0.01$
$P(X \leq 3) = 0.0176 > 0.01$
Answer: $X \leq 2$

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8 Combining two inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x satisfying both

$2x - 2 > 6$ and $x^2 - 5x < 0$.

$2x - 2 > 6$ gives $x > 4$.

$x^2 - 5x < 0$: $(x + 0)(x - 5) < 0$, so $0 < x < 5$.

Both are true where the two ranges overlap.

Answer: $4 < x < 5$

9 Catching up: constant speed against acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes a point O at constant speed 8 m s^{-1}.

At that instant car B starts from rest at O with constant acceleration 4 m s^{-2} in the same direction. Find the time when B catches up A.

Distances equal: $8\,t = \frac{1}{2} \times 4 \times t^2$

$t(8 - 2\,t) = 0$, so $t = 0$ or $t = 8 \div 2$

Answer: 4 s

10 Integration by substitution (All boards)

Use the substitution $u = x^2 + 2$ to find

$\int 2x\,(x^2 + 2)^3\,dx$.

$u = x^2 + 2$, so $du = 2x\,dx$

$\int 1\,u^3\,du = \frac{1\,u^4}{4}$

Replace u: the answer below

Answer: $\frac{1(x^2 + 2)^4}{4} + c$