Day 16 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Standard deviation from summary data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 8 values, $\Sigma x = 184$ and $\Sigma x^2 = 4344$.
Find the mean and the standard deviation.

Mean = $184 \div 8 = 23$

Variance = $\frac{4344}{8} - 23^2 = 543 - 529 = 14$

Standard deviation = $\sqrt{14}$

Answer: mean = 23, standard deviation = 3.74

2 Midpoint and length of a line segment (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

M(−7, −4) is the midpoint of the line segment AB.
A is the point (−3, −1). Find the coordinates of B.

x-coordinate: $\frac{-3 + x_B}{2} = -7$, so $x_B = -11$

y-coordinate: $\frac{-1 + y_B}{2} = -4$, so $y_B = -7$

Answer: B(−11, −7)

3 Solve trig equation (calculator, degrees) (All boards)

Solve $\tan x = 1.50$ for $0° \leq x \leq 360°$.
Give your answers to 1 decimal place.

Reference angle: $\tan^{-1}(1.50) = 56.3°$
tan is positive in the first and third quadrants.
$x = 56.3°$ or $x = 236.3°$
Answer: x = 56.3°, 236.3°

4 Velocity zero (quadratic in t) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds is $v = (t^2 - 7t + 6)$ m s^{-1}.
(a) Find the times when the particle is at rest.
(b) Find its acceleration at the first of these times.

(a) $t^2 - 7t + 6 = 0$
$(t - 1)(t - 6) = 0$

(b) $a = \frac{dv}{dt} = 2t - 7$; at $t = 1$, $a = 2 - 7$

Answer: (a) t = 1 and t = 6 (b) −5 m s^{-2}

5 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Prove, by considering the possible remainders, that the square of any integer is of the form 5k, 5k + 1 or 5k + 4.

Any integer n has the form 5k + r where r = 0, 1, ..., 4.
$(5k)^2 = 25k^2 = 5(5k^2)$
$(5k + 1)^2 = 25k^2 + 10k + 1 = 5(5k^2 + 2k) + 1$
$(5k + 2)^2 = 25k^2 + 20k + 4 = 5(5k^2 + 4k) + 4$
$(5k + 3)^2 = 25k^2 + 30k + 9 = 5(5k^2 + 6k + 1) + 4$
$(5k + 4)^2 = 25k^2 + 40k + 16 = 5(5k^2 + 8k + 3) + 1$
Every case is of the required form.
Answer: Shown

6 Separable differential equations (All boards)

Given that $dy/dx = 4x^2$ and $y = 0$ when $x = 0$,
find y in terms of x.

$y = \int 4x^2\,dx = \frac{4x^3}{3} + c$

$x = 0, y = 0$: $c = 0$

Answer: $y = \frac{4x^3}{3}$

7 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-x^2 + 2x < 0$.

Solve $-x^2 + 2x = 0$: $-(x + 0)(x - 2) = 0$
Critical values $x = 0$ and $x = 2$.
The graph is an ∩-shaped parabola, so take the regions outside the roots.
Answer: $x < 0$ or $x > 2$

8 Binomial: modelling conditions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X is the number of the 7 members of one family who have the flu, which spreads easily.
Can X be modelled by a binomial distribution? Give a reason.

A binomial model needs: a fixed number of trials, two outcomes, constant probability, independent trials.
Here the people are not independent.
Answer: No, the people are not independent

9 Rough slope: limiting equilibrium or sliding (All boards)

A particle slides down a rough plane inclined at angle α to the horizontal, where $\sin\alpha = \frac{4}{5}$ and the coefficient of friction is 0.25. Take $g = 9.8$ m s^{-2}. Find the acceleration.

$R = mg\cos\alpha = m \times 9.8 \times \frac{3}{5}$

$mg\sin\alpha - \mu R = ma$

$a = 9.8 \times \frac{4}{5} - 0.25 \times 9.8 \times \frac{3}{5}$

Answer: 6.37 m s^{-2}

10 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $y^2 = 2x + 7$.
Find dy/dx in terms of x and y, and hence the gradient at (1, 3).

Differentiate both sides with respect to x (using dy/dx for y terms)

$dy/dx = \frac{2}{2y}$

At (1, 3): $dy/dx = \frac{1}{3}$

Answer: $dy/dx = \frac{2}{2y}$; gradient $\frac{1}{3}$