Day 159 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Natural logs and e^x (All boards)

Solve 3 ln x = 6.
Give your answer to 3 significant figures.

ln x = 2
$x = e^2 = 7.39$
Answer: x = 7.39

2 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sequence is defined by $u_{n+2} = u_{n+1} + u_n$, with
$u_1 = 4$ and $u_2 = 1$. Find u_6.

$u_3 = 1 + 4 = 5$
$u_4 = 5 + 1 = 6$, $u_5 = 6 + 5 = 11$
$u_6 = 11 + 6 = 17$
Answer: $u_6 = 17$

3 Inverse trig functions (Edexcel, OCR)

Find the exact value, in radians, of
(a) $\arcsin(\frac{1}{2})$ (b) $\arcsin(-1)$

(a) $y = \arcsin(\frac{1}{2})$ means $\sin y = \frac{1}{2}$ with $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$: $y = \frac{\pi}{6}$

(b) $y = \arcsin(-1)$ means $\sin y = -1$ with $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$: $y = -\frac{\pi}{2}$

Answer: (a) $\frac{\pi}{6}$ (b) $-\frac{\pi}{2}$

4 Position of a point relative to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $(x - 2)^2 + (y - 4)^2 = 9$.
Determine whether the point P(8, −8) is inside, outside or on the circle.

Distance^2 from the centre (2, 4) to P:
$(8 - 2)^2 + (-8 - 4)^2 = 36 + 144 = 180$
$r^2 = 9$, and $180 > 9$.
Answer: P is outside the circle

5 Angle between vectors (scalar product) (CIE)

$\mathbf{a} = 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = -3\mathbf{i} - \mathbf{j} - 3\mathbf{k}$.
Find the angle between a and b, to 1 decimal place.

$\mathbf{a}.\mathbf{b} = (0)(-3) + (3)(-1) + (4)(-3) = -15$
$|\mathbf{a}| = \sqrt{25}$, $|\mathbf{b}| = \sqrt{19}$
$\cos\theta = \frac{-15}{\sqrt{25} \times \sqrt{19}}$, so $\theta = 133.5°$
Answer: 133.5°

6 Continuous random variable: finding k (CIE)

The continuous random variable X has probability density function $f(x) = kx$ for $0 \leq x \leq 2$, and 0 otherwise.
Find the value of k.

$\int$ from 0 to 2 of $k\,x\,dx = 1$
$k \times \frac{2^2}{2} = 1$
Answer: $k = \frac{1}{2}$

Day 159 **A Level Maths: C Tier 10-a-day: Model Solutions**

7 Friction: acceleration along rough ground (All boards)

A block of mass 10 kg is pulled along rough horizontal ground by a horizontal force of 64.2 N.
The coefficient of friction is 0.4. Take g = 9.8 m s^{-2}. Find the acceleration.

R = 10 × 9.8 = 98
F = μR = 0.4 × 98 = 39.2
P − F = ma: 64.2 − 39.2 = 10 a
Answer: 2.5 m s^{-2}

8 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line y = −x + k is a tangent to the curve
y = x^2 − 7x − 3.
Find the value of k.

x^2 − 7x − 3 = −x + k
x^2 − 6x + (−3 − k) = 0
A tangent touches once: discriminant = 0
$(-6)^2$ − 4(−3 − k) = 0
Answer: k = −12

9 Normal approximation to the binomial (CIE)

X ~ B(100, 0.4). Use a normal approximation to find P(X ≥ 38), to 3 s.f.

np = 40 and npq = 24, so use Y ~ N(40, 24)
Continuity correction: P(Y > 37.5)
$z = (37.5 - 40) \div \sqrt{24}$
Answer: 0.695

10 Projectile launched with velocity components (AQA, Edexcel, OCR, MEI)

A ball is projected from level ground with velocity (14**i** + 19.6**j**) m s^{-1} (**i** horizontal, **j** vertically up). Take g = 9.8 m s^{-2}.
(a) Find the time of flight.
(b) Find the greatest height reached.

(a) Vertically: $0 = 19.6\,t - 4.9t^2$
$t = \frac{19.6}{4.9} = 4$
(b) $0 = 19.6^2 - 2 \times 9.8 \times H$
$H = \frac{384.16}{19.6}$
Answer: (a) 4 s (b) 19.6 m