Day 156 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 F = ma: find the acceleration (All boards)

A horizontal force of 119 N acts on a crate of mass 10 kg on a smooth horizontal surface.
Find the acceleration.

$F = ma$
$119 = 10\ a$
Answer: 11.9 m s^{-2}

2 Trapezium rule from a table (AQA, Edexcel, OCR, MEI, Eduqas)

The table shows values of y = f(x).
x: 1 1.25 1.5 1.75 2 2.25
y: 7 2.6 4.2 2.2 3.8 2.1
Use the trapezium rule to estimate the area under the curve between x = 1 and x = 2.25.

h = 0.25 with 5 strips
$\text{Area} \approx \frac{0.25}{2}[7 + 2.1 + 2(2.6 + 4.2 + 2.2 + 3.8)]$
$= \frac{0.25}{2} \times 34.7$
$= 4.3375$
Answer: 4.3375

3 The remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the remainder when $-x^3 - x^2 + 5x + 7$ is divided by $(x - 1)$.

Remainder = f(1)
$= (-1) + (-1) + 5 + 7$
$= 10$
Answer: 10

4 Discrete random variable: E(X) and Var(X) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The discrete random variable X has this distribution.
x: 3, 4, 5, 6
P(X = x): 0.3, 0.3, 0.1, 0.3
Find E(X) and Var(X).

$E(X) = \Sigma xp = 3\times0.3 + 4\times0.3 + 5\times0.1 + 6\times0.3 = 4.4$
$E(X^2) = \Sigma x^2 p = 20.8$
$\text{Var}(X) = 20.8 - 4.4^2$
Answer: E(X) = 4.4, Var(X) = 1.44

5 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|x - 2| \geq 4$.

$x - 2 \geq 4$ or $x - 2 \leq -4$
Add 2 to each side.
Answer: $x \leq -2$ or $x \geq 6$

6 Product rule (All boards)

Use the product rule to differentiate
$y = x^3 \sin 3x$.

$u = x^3$, $v = \sin 3x$
$u' = 3x^2$, $v' = 3\cos 3x$
$dy/dx = u'v + uv'$, which simplifies to the answer below
Answer: $dy/dx = x^2(3 \sin 3x + 3x \cos 3x)$

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7 Frequency density (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In a histogram, the class 0 to 5 has frequency 25 and is drawn with a bar of height 4 cm. The class 5 to 15 has frequency 30. Find the height of its bar.

Frequency density = frequency ÷ class width
First bar: 25 ÷ 5 = 5, so 4 cm represents 5
Second bar: 30 ÷ 10 = 3

Height = $4 \times \frac{3}{5}$

Answer: 2.4 cm

8 Velocity zero (quadratic in t) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds is $v = (t^2 - 6t + 5)$ m s^{-1}.
(a) Find the times when the particle is at rest.
(b) Find its acceleration at the first of these times.

(a) $t^2 - 6t + 5 = 0$
$(t - 1)(t - 5) = 0$

(b) $a = \frac{dv}{dt} = 2t - 6$; at $t = 1$, $a = 2 - 6$

Answer: (a) t = 1 and t = 5 (b) −4 m s^{-2}

9 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 - 3x)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots$ with $n = \frac{1}{2}$, $y = -3x$

$= 1 + \frac{1}{2}(-3x) + \frac{\frac{1}{2}((-\frac{1}{2}))}{2}(-3x)^2$

$= 1 - \frac{3}{2}x - \frac{9}{8}x^2$

Valid when $|-3x| < 1$, i.e. $|x| < \frac{1}{3}$

Answer: $1 - \frac{3}{2}x - \frac{9}{8}x^2$; $|x| < \frac{1}{3}$

10 Optimisation (algebraic set-up) (All boards)

A farmer has 30 m of fencing for three sides of a rectangular pen; the fourth side is a wall. Find the maximum area of the pen.

Two sides of length x m: $A = x(30 - 2x) = 30x - 2x^2$

$dA/dx = 30 - 4x = 0$, so $x = \frac{15}{2}$

$d^2A/dx^2 = -4 < 0$ (maximum); $A = \frac{15}{2} \times 15$

Answer: $\frac{225}{2}$ m²