Day 155 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Integrate (ax + b)^n and related (All boards)

Find $\int (2x + 1)^4 \, dx$.

Reverse the chain rule: raise the power by 1, divide by the new power and by 2

Here a = 2

Answer: $\frac{(2x + 1)^5}{10} + c$

2 Equation of a circle from a diagram (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a circle with the centre marked.

The circle touches the x-axis. Find the equation of the circle.

The circle touches the x-axis, so the radius equals the y-coordinate of the centre: r = 3.

Centre (3, 3): $(x - a)^2 + (y - b)^2 = r^2$

Answer: $(x - 3)^2 + (y - 3)^2 = 9$

3 Solve basic trig equation (exact, degrees) (All boards)

Solve $\sin x = \frac{\sqrt{3}}{2}$ for $0° \le x \le 360°$.

Reference angle: $\sin 60° = \frac{\sqrt{3}}{2}$

sin is positive in the first and second quadrants.

x = 60°, 120°

Answer: x = 60°, 120°

4 Inclined plane: normal reaction and holding force (All boards)

A particle of mass 15 kg rests on a smooth plane inclined at α to the horizontal, where $\tan \alpha = \frac{3}{4}$.

It is held by a force P N up the slope. Take g = 9.8 m s^{-2}. Find P and the normal reaction.

Parallel: $P = mg \sin \alpha = 15 \times 9.8 \times \frac{3}{5}$

Perpendicular: $R = mg \cos \alpha = 15 \times 9.8 \times \frac{4}{5}$

Answer: P = 88.2, R = 117.6

5 Frequency density (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In a histogram, the class 30 to 35 has frequency 50 and is drawn with a bar of height 4 cm.

The class 35 to 55 has frequency 35. Find the height of its bar.

Frequency density = frequency ÷ class width

First bar: 50 ÷ 5 = 10, so 4 cm represents 10

Second bar: 35 ÷ 20 = 1.75

Height = $4 \times \frac{1.75}{10}$

Answer: 0.7 cm

6 Vector equation of a line (3D) (CIE)

A line has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix} + t\begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$.

Write down a vector equation of the line through the origin, parallel to this line.

Parallel lines have the same direction vector

The line passes through O, so the position vector is the zero vector

Answer: $\mathbf{r} = s\begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$

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7 Finding a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve $y = f(x)$, where $f(x)$ is a cubic. Find $f(x)$ in the form $k(x - a)(x - b)(x - c)$.

The curve crosses the x-axis at $x = -4$, -1 and 1.
$f(x) = k(x + 4)(x + 1)(x - 1)$
At $(0, 8)$: $k \times 4 \times 1 \times (-1) = 8$
$k = -2$
Answer: $f(x) = -2(x + 4)(x + 1)(x - 1)$

8 Vertical motion under gravity: thrown upwards (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A rock is thrown vertically upwards with speed 49 m s^{-1}. Take $g = 9.8$ m s^{-2}.
(a) Find the time taken to reach its greatest height.
(b) Find the greatest height reached.

(a) $v = u + at$: $0 = 49 - 9.8t$, so $t = 5$
(b) $v^2 = u^2 + 2as$: $0 = 49^2 - 2 \times 9.8 \times s$
$s = \frac{2401}{19.6}$
Answer: (a) 5 s (b) 122.5 m

9 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that it rains is 0.3. If so, the probability that Sam is late is 0.6; otherwise it is 0.3.
Given that Sam is late, find the probability that it rains.

P(both) = $0.3 \times 0.6 = 0.18$; P(not first and second) = $0.7 \times 0.3 = 0.21$
P(second) = $0.18 + 0.21 = 0.39$
P = $0.18 \div 0.39$
Answer: $\frac{6}{13} = 0.462$

10 Real roots and the discriminant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the range of values of k for which $kx^2 + 8x + 5 = 0$ has real roots.

Real roots: $b^2 - 4ac \geq 0$
$64 - 20k \geq 0$, so $k \leq \frac{16}{5}$
Also $k \neq 0$ for the equation to be a quadratic.
Answer: $k \leq \frac{16}{5}$, $k \neq 0$