Day 154 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

The velocity of a boat is $(-12\mathbf{i} + 5\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors.
(a) Find the speed.
(b) Find the acute angle between the direction of motion and the line of $\mathbf{i}$. Give 1 d.p.

(a) speed $= \sqrt{12^2 + 5^2} = \sqrt{169}$

(b) $\tan\theta = \frac{5}{12}$

$\theta = \tan^{-1}(\frac{5}{12})$

Answer: (a) 13 m s^{-1} (b) 22.6°

2 Solve basic trig equation (exact, degrees) (All boards)

Solve $\cos x = -\frac{\sqrt{3}}{2}$ for $-180° \leq x \leq 180°$.

Reference angle: $\cos 30° = \frac{\sqrt{3}}{2}$

cos is negative in the second and third quadrants.
Using the cos graph/CAST and the interval $-180° \leq x \leq 180°$:
$x = -150°, 150°$
Answer: $x = -150°, 150°$

3 Intercepts of a cubic graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = -(x + 4)(x - 1)(x - 2)$. Write down the coordinates of the points where the curve meets the axes.

$y = 0$: $x = -4, 1, 2$
$x = 0$: $y = 4 \times (-1) \times (-2) \times (-1) = -8$
Answer: (−4, 0), (1, 0), (2, 0), (0, −8)

4 Moments: balancing a plank (AQA, Edexcel, OCR, MEI)

A light plank rests on a pivot. A child of mass 45 kg sits 2.5 m from the pivot on one side. Where must a child of mass 25 kg sit on the other side for the plank to balance horizontally?

Moments about the pivot:
$45\text{ g} \times 2.5 = 25\text{ g} \times d$
$d = \frac{45 \times 2.5}{25}$
Answer: 4.5 m from the pivot

5 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Prove, by considering the possible remainders, that the square of any integer is of the form 3k or 3k + 1.

Any integer n has the form $3k + r$ where $r = 0, 1, ..., 2$.
$(3k)^2 = 9k^2 = 3(3k^2)$
$(3k + 1)^2 = 9k^2 + 6k + 1 = 3(3k^2 + 2k) + 1$
$(3k + 2)^2 = 9k^2 + 12k + 4 = 3(3k^2 + 4k + 1) + 1$
Every case is of the required form.
Answer: Shown

6 Integration by substitution (All boards)

Use $u = x^2 + 1$ to evaluate

$\int_0^2 2x(x^2 + 1)^3\,dx$.

$u = x^2 + 1$, $du = 2x\,dx$; $x = 0$ gives $u = 1$, $x = 2$ gives $u = 5$

$\int_1^5 u^3\,du = [\frac{u^4}{4}]$ from 1 to 5

$= \frac{5^4 - 1^4}{4}$

Answer: 156

Day 154 **A Level Maths: C Tier 10-a-day: Model Solutions**

7 Midpoint and length of a line segment (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

M(−3, 6) is the midpoint of the line segment AB.
A is the point (−4, 5). Find the coordinates of B.

x-coordinate: $\frac{-4 + x_B}{2} = -3$, so $x_B = -2$

y-coordinate: $\frac{5 + y_B}{2} = 6$, so $y_B = 7$

Answer: B(−2, 7)

8 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(30, p). H_0: p = 0.2, H_1: p > 0.2.
Find the critical region for a test at the 1% significance level.

$P(X \geq 12) = 0.00949 \leq 0.01$
$P(X \geq 11) = 0.0256 > 0.01$
Answer: X ≥ 12

9 Amplitude and period from a graph (diagram) (All boards)

The diagram shows the graph of y = a sin bx for 0° ≤ x ≤ 360°, where a and b are positive integers.
Find a and b.

The maximum value of y is 2, so a = 2

The graph completes 4 full cycles in 360°, so the period is 90° and $b = \frac{360}{90} = 4$

Answer: a = 2, b = 4

10 Selection without replacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A bag has 6 red and 2 blue counters. Two are taken at random without replacement.
Find the probability that at least one of them is red.

$P(\text{no red}) = P(BB) = \frac{2}{8} \times \frac{1}{7} = \frac{1}{28}$

$P(\text{at least one red}) = 1 - \frac{1}{28}$

Answer: $\frac{27}{28}$