Day 153 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Binomial expansion: positive integer powers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^3 in the expansion of $(3 - x)^8$.

Term in x^3: ${}^8C_3 \times 3^5 \times (-1\,x)^3$

$= 56 \times 243 \times -1$

Answer: -13608

2 Box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plot summarises the journey times, in minutes, of some commuters.
Find the range and the interquartile range.

Range $= 55 - 15 = 40$

IQR $= 50 - 40 = 10$

Answer: range = 40, IQR = 10

3 Solve a^x = b using logarithms (All boards)

Solve $6^{x+3} = 200$.
Give your answer to 3 significant figures.

Take logs: $(x + 3) \log 6 = \log 200$

$x + 3 = 2.957$, so $x = -0.0430$

Answer: $x = -0.0430$

4 Velocity-time graph: acceleration from two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The graph shows the velocity of a skater.
(a) Find the acceleration in the first 8 s.
(b) Find the deceleration in the last 4 s.
(c) Find the total distance travelled.

(a) $a = \dfrac{10 - 6}{8}$

(b) $a = \dfrac{10 - 0}{4}$, a deceleration of 2.5

(c) area $= \frac{1}{2}(6 + 10) \times 8 + 10 \times 10 + \frac{1}{2} \times 4 \times 10$

Answer: (a) 0.5 m s^{-2} (b) 2.5 m s^{-2} (c) 184 m

5 Newton-Raphson: one or two steps (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^3 - x - 3$.
Use the Newton-Raphson method with $x_0 = 2$ to find x_1 and x_2, giving your answer to 3 decimal places.

$f'(x) = 3x^2 - 1$

$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$x_1 = 2 - \dfrac{3.000}{11.00} = 1.7273$

$x_2 = 1.7273 - \dfrac{0.4260}{7.950} = 1.6737$

Answer: $x_1 = 1.727$, $x_2 = 1.674$

6 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-2x - 8 > -6$.

$-2x > 2$

Divide by -2 and reverse the inequality:

$x < -1$

Answer: $x < -1$

7 Solve a logarithmic equation (two logs) (All boards)

Solve $\log x + \log (x - 2) = \log 8$, $x > 2$.

$\log [x(x - 2)] = \log 8$, so $x^2 - 2x = 8$

$x^2 - 2x - 8 = 0$, $(x - 4)(x + 2) = 0$

$x = 4$ or $x = -2$; $x > 2$, so $x = 4$

Answer: $x = 4$

8 Binomial: at least one success (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Each customer buys a coffee with probability 0.3.

Find the probability that at least one of 7 customers buys a coffee, to 3 s.f.

$P(\text{zero}) = 0.7^7$

$P(\text{at least one}) = 1 - 0.7^7$

Answer: 0.918

9 Vertical motion under gravity: thrown upwards (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is thrown vertically upwards with speed 39.2 m s^{-1}. Take $g = 9.8$ m s^{-2}.

(a) Find the time taken to reach its greatest height.

(b) Find the greatest height reached.

(a) $v = u + at$: $0 = 39.2 - 9.8t$, so $t = 4$

(b) $v^2 = u^2 + 2as$: $0 = 39.2^2 - 2 \times 9.8 \times s$

$s = \frac{1536.64}{19.6}$

Answer: (a) 4 s (b) 78.4 m

10 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 + 3x)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n - 1)}{2!}y^2 + \ldots$ with $n = \frac{1}{2}$, $y = 3x$

$= 1 + \frac{1}{2}(3x) + \frac{\frac{1}{2}((-\frac{1}{2}))}{2}(3x)^2$

$= 1 + \frac{3}{2}x - \frac{9}{8}x^2$

Valid when $|3x| < 1$, i.e. $|x| < \frac{1}{3}$

Answer: $1 + \frac{3}{2}x - \frac{9}{8}x^2$; $|x| < \frac{1}{3}$