Day 152 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 F = ma: find the acceleration (All boards)

A horizontal force of 114 N acts on a sledge of mass 3 kg on a smooth horizontal surface. Find the acceleration.

$F = ma$

$114 = 3\,a$

Answer: 38 m s^{-2}

2 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\sqrt{4x^6} \times 3x$.

$\sqrt{4x^6} = (4x^6)^{\frac{1}{2}} = 2\,x^3$

$2\,x^3 \times 3x = 6x^4$

Answer: $6x^4$

3 Selection without replacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A bag has 7 red and 7 blue marbles. Two are taken at random without replacement. Find the probability that at least one of them is red.

$\text{P(no red)} = \text{P(BB)} = \frac{7}{14} \times \frac{6}{13} = \frac{3}{13}$

$\text{P(at least one red)} = 1 - \frac{3}{13}$

Answer: $\frac{10}{13}$

4 Change of sign to locate a root (All boards)

$f(x) = x^2 - \ln x - 5$.

Show that $f(x) = 0$ has a root between $x = 2$ and $x = 3$.

$f(2) = -1.69$

$f(3) = 2.90$

f is continuous and there is a change of sign, so a root lies in the interval

Answer: f(2) = −1.69, f(3) = 2.90: change of sign

5 Moments: reactions on a uniform beam (AQA, Edexcel, OCR, MEI)

A uniform beam AB of mass 40 kg and length 6 m rests horizontally on supports at A and B. A load of mass 10 kg is placed on the beam 1.5 m from A. Take g = 9.8 m s^{-2}. Find the reaction at B.

Moments about A:

$R_B \times 6 = 40 \times 9.8 \times 3 + 10 \times 9.8 \times 1.5$

$R_B \times 6 = 1323$

Answer: 220.5 N

6 Partial fractions: find the constants (All boards)

Express $\frac{x+6}{(x+3)(x+2)}$ in the form $\frac{A}{x+3} + \frac{B}{x+2}$.

Find the values of A and B.

$x + 6 \equiv A(x+2) + B(x+3)$

$x = -3$: $3 = A(-1)$, so $A = -3$

$x = -2$: $4 = B(1)$, so $B = 4$

Answer: A = −3, B = 4

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7 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 6^2)$. H_0: $\mu = 40$, H_1: $\mu < 40$.

A sample of 16 has mean $\bar{x} = 38.5$.
Test at the 5% level. Critical value: $z = -1.645$.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (38.5 - 40) \div (6 \div 4)$
$z = -1.00$
-1.00 is outside the critical region, so do not reject H_0
Answer: $z = -1.00$; do not reject H_0

8 Modulus graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = |2x + 5|$. Write down the coordinates of the vertex and of the point where the graph meets the y-axis.

The vertex is on the x-axis where $2x + 5 = 0$: $x = -\frac{5}{2}$

$x = 0$: $y = |5| = 5$

Answer: Vertex $(-\frac{5}{2}, 0)$; y-axis $(0, 5)$

9 Collinear points using vectors (All boards)

A$(-2, -1)$, B$(4, 2)$ and C$(8, 4)$.
Show that A, B and C are collinear and find AB : BC.

$\text{AB} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$ and $\text{BC} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

$\text{AB} = 3\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\text{BC} = 2\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, so AB and BC are parallel with common point B

Hence A, B, C are collinear and AB : BC = 3 : 2
Answer: collinear; AB : BC = 3 : 2

10 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -2x + k$ is a tangent to the curve
$y = x^2 + 4$.
Find the value of k.

$x^2 + 4 = -2x + k$
$x^2 + 2x + (4 - k) = 0$
A tangent touches once: discriminant = 0
$(2)^2 - 4(4 - k) = 0$
Answer: $k = 3$