Day 151 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sequence is defined by $u_{n+1} = \frac{1}{3} u_n + 6$.

The sequence converges to a limit L. Find L.

As $n \to \infty$, u_{n+1} and u_n both tend to L.

$L = \frac{1}{3} L + 6$

$L(1 - \frac{1}{3}) = 6$

Answer: L = 9

2 Regression: prediction and reliability (AQA, Edexcel, OCR, MEI, Eduqas)

For 7 data pairs, with advertising spend (£s hundreds) from 10 to 20, the regression line is S = 4 + 7a.
Estimate sales (£s thousands) when a = 24 and comment on the reliability.

$S = 4 + 7 \times 24 = 172$

24 is outside the range 10 to 20, so this is extrapolation.

Answer: S = 172; estimate is unreliable, as 24 is outside the data range (extrapolation)

3 Constant acceleration: distance from average speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car moving in a straight line with constant acceleration increases its speed from 1 m s^{-1} to 5 m s^{-1} in 9 s.
Find the distance it travels in this time.

$s = \frac{(u + v)}{2} t$

$s = \frac{(1 + 5)}{2} \times 9$

Answer: 27 m

4 Discriminant: equal roots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$x^2 + kx + 49 = 0$ has equal roots.
Find the possible values of k.

Equal roots: $b^2 - 4ac = 0$

$k^2 - 4 \times 1 \times 49 = 0$, so $k^2 = 196$

Answer: k = ±14

5 Solve with a shifted angle (All boards)

Solve $\cos(x - 20°) = -\frac{1}{2}$ for $0° \leq x \leq 360°$.

Let $u = x - 20°$, so $-20° \leq u \leq 340°$

$\cos u = -\frac{1}{2}$: $u = 120°, 240°$

$x = u + 20°$: $x = 140°, 260°$

Answer: x = 140°, 260°

6 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(25, p)$. H_0: $p = 0.4$, H_1: $p > 0.4$.

Find the critical region for a test at the 5% significance level.

$P(X \geq 15) = 0.0344 \leq 0.05$

$P(X \geq 14) = 0.0778 > 0.05$

Answer: X ≥ 15

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7 Projectile launched with velocity components (AQA, Edexcel, OCR, MEI)

A ball is projected from level ground with velocity $(5\mathbf{i} + 9.8\mathbf{j})$ m s^{-1} ($\mathbf{i}$ horizontal, $\mathbf{j}$ vertically up). Take g = 9.8 m s^{-2}.
(a) Find the time of flight.
(b) Find the greatest height reached.

(a) Vertically: $0 = 9.8\,t - 4.9t^2$

$t = \frac{9.8}{4.9} = 2$

(b) $0 = 9.8^2 - 2 \times 9.8 \times H$

$H = \frac{96.04}{19.6}$

Answer: (a) 2 s (b) 4.9 m

8 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$x = 4t^2 + 3$, $y = 2t^3 - t$.
Find dy/dx in terms of t, and its value when t = 1.

$dx/dt = 8t$, $dy/dt = 6t^2 - 1$

$dy/dx = \frac{dy}{dt \div dx/dt} = \frac{6t^2 - 1}{8t}$

At t = 1: $dy/dx = \frac{5}{8}$

Answer: $dy/dx = \frac{6t^2 - 1}{8t}$; $\frac{5}{8}$

9 Linear and quadratic simultaneous equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations:
$y = x^2 + x + 6$, $y = 2x + 6$.

$x^2 + x + 6 = 2x + 6$
$x^2 - x = 0$
$(x + 0)(x - 1) = 0$
Use $y = 2x + 6$ to find each y.
Answer: x = 0, y = 6 and x = 1, y = 8

10 Solve trig equation in radians (calculator) (All boards)

Solve $4 \tan x = -9$ for $0 \leq x \leq 2\pi$.
Give your answers to 3 significant figures.

$\tan x = -\frac{9}{4}$

Reference angle = $\tan^{-1}(2.250) = 1.153$ radians
tan is negative in the second and fourth quadrants.
x = 1.99, 5.13
Answer: x = 1.99, 5.13