Day 148 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Change of sign to locate a root (All boards)

$f(x) = x^3 - 4x - 8$.

Show that $f(x) = 0$ has a root between $x = 2$ and $x = 3$.

$f(2) = -8.00$

$f(3) = 7.00$

f is continuous and there is a change of sign, so a root lies in the interval

Answer: f(2) = −8.00, f(3) = 7.00: change of sign

2 Lift: tension in the cable (All boards)

A lift of mass 400 kg is raised by a vertical cable with constant acceleration 0.75 m s^{-2} upwards. Take g = 9.8 m s^{-2}. Find the tension in the cable.

Resolving upwards: $T - 400 \times 9.8 = 400 \times 0.75$

$T = 3920 + 300$

Answer: 4220 N

3 Laws of logarithms: single log (All boards)

Write $5 \log 2 + \log 5 - \log 10$ as a single logarithm, giving it in the form $\log N$ where N is an integer.

$5 \log 2 = \log 2^5 = \log 32$

$\log 32 + \log 5 - \log 10 = \log \frac{32 \times 5}{10} = \log 16$

Answer: log 16

4 Reciprocal graphs and asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = \frac{-6}{x - 4} + 2$.

Write down the equations of the asymptotes and the coordinates where the curve meets the axes.

Asymptotes: $x = 4$ and $y = 2$.

$y = 0$: $\frac{-6}{x - 4} = -2$, so $x = 7$

$x = 0$: $y = \frac{-6}{-4} + 2 = \frac{7}{2}$

Answer: x = 4, y = 2 ; (7, 0), (0, $\frac{7}{2}$)

5 Constant acceleration: distance from average speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car moving in a straight line with constant acceleration increases its speed from 10 m s^{-1} to 26 m s^{-1} in 10 s. Find the distance it travels in this time.

$s = \frac{(u + v)}{2} t$

$s = \frac{(10 + 26)}{2} \times 10$

Answer: 180 m

6 Regression: interpreting coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

A model relates the amount, y litres, left in a tank to the time, t hours, since it was filled by $y = 54 - 8.5t$. Interpret the value 54 in this context.

The constant is the predicted value of y when $t = 0$.

So the predicted y at $t = 0$ is 54 litres.

Answer: predicted y is 54 litres when t = 0

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7 Finding a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve $y = f(x)$, where $f(x)$ is a cubic. Find $f(x)$ in the form $k(x - a)(x - b)(x - c)$.

The curve crosses the x-axis at $x = -3, -2$ and 2.
$f(x) = k(x + 3)(x + 2)(x - 2)$
At (0, 24): $k \times 3 \times 2 \times (-2) = 24$
$k = -2$
Answer: $f(x) = -2(x + 3)(x + 2)(x - 2)$

8 Optimisation (algebraic set-up) (All boards)

A rectangle has perimeter 40 cm.
Find the maximum possible area of the rectangle and the dimensions that give it.

Sides x and $20 - x$, so $A = x(20 - x) = 20x - x^2$
$dA/dx = 20 - 2x = 0$, so $x = 10$
$d^2A/dx^2 = -2 < 0$, so it is a maximum; $A = 10 \times 10$
Answer: $A = 100$ cm^2, a 10 cm square

9 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a patient takes the drug is 0.3. If so, the probability that the patient recovers is 0.6; otherwise it is 0.2. Given that the patient recovers, find the probability that a patient takes the drug.

P(both) = $0.3 \times 0.6 = 0.18$; P(not first and second) = $0.7 \times 0.2 = 0.14$
P(second) = $0.18 + 0.14 = 0.32$
$P = 0.18 \div 0.32$
Answer: $\frac{9}{16} = 0.5625$

10 Intercepts of a cubic graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = 2(x + 4)(x - 1)(x - 4)$. Write down the coordinates of the points where the curve meets the axes.

$y = 0$: $x = -4, 1, 4$
$x = 0$: $y = 4 \times (-1) \times (-4) \times 2 = 32$
Answer: (−4, 0), (1, 0), (4, 0), (0, 32)