Day 147 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Selection without replacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A bag has 5 red and 2 blue sweets. Two are taken at random without replacement.
Find the probability that they are both the same colour.

P(RR) = $\frac{5}{7} \times \frac{4}{6}$, P(BB) = $\frac{2}{7} \times \frac{1}{6}$

Add: $\frac{10}{21} + \frac{1}{21}$

Answer: $\mathbf{\frac{11}{21}}$

2 Geometric sequences (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In a geometric sequence $u_3 = 24$ and $u_4 = 48$.
Find the first term and the common ratio.

$r = u_4 \div u_3 = 48 \div 24 = 2$

$a = u_3 \div r^2 = 6$

Answer: a = 6, r = 2

3 Laws of logarithms: single log (All boards)

Write 2 log 2 + log 3 − log 2 as a single logarithm,
giving it in the form log N where N is an integer.

$2 \log 2 = \log 2^2 = \log 4$

$\log 4 + \log 3 - \log 2 = \log \frac{4 \times 3}{2} = \log 6$

Answer: log 6

4 Car towing a caravan (All boards)

A car of mass 1500 kg tows a caravan of mass 500 kg on a level road with acceleration 0.75 m s^{-2}. Resistances: 400 N on the car, 100 N on the caravan. Find the driving force and the tow bar tension.

Whole system: D − 400 − 100 = 2000 × 0.75
Caravan: T − 100 = 500 × 0.75
Answer: D = 2000 N, T = 475 N

5 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

f(x) = 6x + 4.
Find $f^{-1}(x)$.

y = 6x + 4, so y − 4 = 6x

$x = \frac{y - 4}{6}$

Answer: $\mathbf{f^{-1}(x) = \frac{x - 4}{6}}$

6 Rewrite a sin x + b cos x in R form (All boards)

Express 7 sin x + 24 cos x in the form R sin(x + α), where R > 0 and 0° < α < 90°.
Give α to 1 decimal place.

R sin(x + α) = R sin x cos α + R cos x sin α
R cos α = 7 and R sin α = 24

$R = \sqrt{7^2 + 24^2} = 25$, $\tan \alpha = \frac{24}{7}$, α = 73.7°

Answer: 25 sin(x + 73.7°)

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7 Finding an unknown using the factor theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$(x + 1)$ is a factor of $x^3 + kx^2 - 3x - 6$.
Find the value of k.

Let $f(x) = x^3 + kx^2 - 3x - 6$. Then $f(-1) = 0$:
$-1 + k - 3 = 0$
$k = 4$
Answer: $k = 4$

8 Two particles over a smooth pulley (All boards)

Two particles of masses 1 kg and 6 kg hang from the ends of a light inextensible string over a smooth pulley and are released from rest. Take $g = 9.8$ m s^{-2}. Find the acceleration and the tension in the string.

6 kg: $6 \times 9.8 - T = 6a$
1 kg: $T - 1 \times 9.8 = 1a$
Adding: $49 = 7a$, then $T = 1(9.8 + a)$
Answer: $a = 7$ m s^{-2}, $T = 16.8$ N

9 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(10, p)$. H_0: $p = 0.4$, H_1: $p < 0.4$.
Find the critical region for a test at the 5% significance level.

$P(X \leq 1) = 0.0464 \leq 0.05$
$P(X \leq 2) = 0.167 > 0.05$
Answer: $X \leq 1$

10 Identity then solve (sin² to cos) (All boards)

Solve $-3\cos^2 x + 3\sin x + 3 = 0$
for $0° \leq x \leq 360°$.

Use $\cos^2 x = 1 - \sin^2 x$: $3\sin^2 x + 3\sin x = 0$
Factorise and solve: $\sin x = -1$ or $\sin x = 0$
$x = 0°, 180°, 270°, 360°$
Answer: $x = 0°, 180°, 270°, 360°$