Day 146 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Divide $f(x) = x^4 + 5x^3 + 2x^2 - x - 6$ by $(x + 1)$.
Find the quotient and the remainder.

Use synthetic division with $a = -1$:
Bottom row: 1, 4, −2, 1, −7
Quotient: $x^3 + 4x^2 - 2x + 1$
Remainder: −7
Answer: Quotient $x^3 + 4x^2 - 2x + 1$, remainder −7

2 Inclined plane: normal reaction and holding force (All boards)

A particle of mass 26 kg rests on a smooth plane inclined at α to the horizontal, where $\tan\alpha = \frac{5}{12}$.

It is held by a force P N up the slope. Take $g = 9.8$ m s^{-2}. Find P and the normal reaction.

Parallel: $P = mg\sin\alpha = 26 \times 9.8 \times \frac{5}{13}$

Perpendicular: $R = mg\cos\alpha = 26 \times 9.8 \times \frac{12}{13}$

Answer: P = 98, R = 235.2

3 Pythagorean identity: find cos and tan (All boards)

θ is acute and $\sin\theta = \frac{5}{13}$.

Find the exact values of $\cos\theta$ and $\tan\theta$.

$\sin^2\theta + \cos^2\theta = 1$, so $\cos^2\theta = 1 - \frac{25}{169} = \frac{144}{169}$

θ is acute, so $\cos\theta = \frac{12}{13}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{5}{12}$

Answer: $\cos\theta = \frac{12}{13}$, $\tan\theta = \frac{5}{12}$

4 Equation of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (−2, 5) and B is (4, 5). AB is a diameter of a circle.
Find the equation of the circle.

Centre = midpoint of AB = (1, 5)
$r^2 = (4 - 1)^2 + (5 - 5)^2 = 9$
Equation: $(x - a)^2 + (y - b)^2 = r^2$
Answer: $(x - 1)^2 + (y - 5)^2 = 9$

5 Car towing a caravan (All boards)

A car of mass 800 kg tows a caravan of mass 500 kg on a level road with acceleration 0.5 m s^{-2}. Resistances: 300 N on the car, 100 N on the caravan. Find the driving force and the tow bar tension.

Whole system: $D - 300 - 100 = 1300 \times 0.5$
Caravan: $T - 100 = 500 \times 0.5$
Answer: D = 1050 N, T = 350 N

6 Binomial: at least one success (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Each time a spinner is spun the probability of red is 0.25.
Find the probability of getting red at least once in 9 spins, to 3 s.f.

P(zero) = 0.75^9
P(at least one) = $1 - 0.75^9$
Answer: 0.925

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7 Finding a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), where f(x) is a cubic. Find f(x) in the form k(x − a)(x − b)(x − c).

The curve crosses the x-axis at x = −2, 1 and 3.
f(x) = k(x + 2)(x − 1)(x − 3)
At (0, 12): k × 2 × (−1) × (−3) = 12
k = 2
Answer: f(x) = 2(x + 2)(x − 1)(x − 3)

8 Area between a line and a curve (All boards)

The curve $y = 5x - x^2$ and the line $y = 2x$ meet at the origin and at P.
Find the area enclosed between them.

$5x - x^2 = 2x$ gives $3x - x^2 = 0$, so $x = 0$ or $x = 3$

Area = $\int_0^3 (3x - x^2)\,dx = \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]$ from 0 to 3

$= \frac{3^3}{2} - \frac{3^3}{3} = \frac{9}{2}$

Answer: $\frac{9}{2}$

9 Binomial: P(X = k) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(18, 0.2). Find P(X = 6), giving your answer to 3 s.f.

$P(X = 6) = {}^{18}C_6 \times 0.2^6 \times 0.8^{12}$

$= 18564 \times 0.2^6 \times 0.8^{12}$

Answer: 0.0816

10 Differentiate tan, sec, cosec, cot (AQA, Edexcel, OCR, MEI, Eduqas)

Differentiate y = sec 4x with respect to x.

Use the standard result: d/dx(sec kx) = k sec kx tan kx
Answer: dy/dx = 4 sec 4x tan 4x